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TWO-DIMENSIONAL COMPRESSIBLE FLOW IN CENTRIFUGAL
COMPRESSORS WITH LOGARITHMIC-SPIRAL BLADES

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SUMMARY

Two numerical examples are presented for two-dimensional, compressible, nonviscous flow in centrifugal compressors with backward-curved, logarithmic-spiral blades, the center lines of which (in the axial-radial plane) generate a right circular cone when rotated about the axis of the compressor. The two examples are for different blade angles and are compared with a third example for straight, radial blades. The solutions were obtained in a region of the compressors (including the impeller tip) that was considered to be unaffected by the impeller-inlet configuration and by the diffuser vanes. The numerical results are presented in tables of the stream-function distribution and in plots of the streamlines, constant Mach number lines, and constant-pressure-ratio lines. In addition, a simplified analysis for logarithmic-spiral blades is presented that can be used to determine flow conditions within the impeller except near the impeller tip and the impeller inlet.

For the same impeller-tip speed, number of blades, and compressor flow rate, it was concluded that: (1) The deceleration along the trailing face of the blade (face opposed to the direction of rotation) is less for backward-curved blades than for straight blades; (2) the relative eddy, which forms on the driving face of the blade at low ratios of flow rate to impeller-tip speed, can be reduced or eliminated by backward-curved or forward-curved blades; (3) the static pressures are essentially equal at the impeller tip for straight and backward-curved blades. The slip factor, corrected for the tangential component of the relative flow assuming perfect guiding of the fluid by the blades, was found to be independent of the blade angle for logarithmic-spiral blades.

INTRODUCTION

Improved performance of centrifugal compressors should result from improved design methods based upon increased knowledge of the complicated flow processes in centrifugal compressors. A series of solutions have therefore been obtained at the NACA Lewis laboratory for two-dimensional, compressible, nonviscous flow through centrifugal compressors. Each solution shows the effect of varying a single design or operating variable from the fixed standard values common to the other solutions. The solutions are not intended to provide specific design information but are meant to clarify the flow phenomena in centrifugal compressors.

A general method of analysis is developed in reference 1 for conical-flow compressors and turbines in which the center line of the passage generates a right circular cone about the axis of rotation. The radial-flow centrifugal compressor is a special case in which the cone angle is 180° .

The general method of analysis is used in reference 2 to investigate the effect of compressor-flow rate, impeller-tip speed, and the number of blades upon the compressible flow within a certain region of radial- or conical-flow compressors with straight blades. The region investigated, which included the impeller tip, was assumed to be unaffected by the impeller-inlet configuration and by the diffuser vanes; that is, the impeller inlet and the diffuser vanes, if any, were considered far enough removed from the region investigated not to appreciably affect the flow in that region.

Solutions are extended herein to two impellers with backward-curved, logarithmic-spiral blades. The solutions are obtained for flow regions of the compressor subject to the same restrictions outlined for the straight-blade solutions. The purpose of these solutions is to investigate the effect of logarithmic-spiral-blade curvature upon the flow conditions within the compressor, especially along the blade surfaces, and to obtain qualitative information on the effect of blade curvature in general. In addition, a simplified method of analysis is presented that permits the relatively rapid calculation of flow conditions within the impeller, except near the impeller tip and impeller inlet and which is used to draw some conclusions with regard to forward-curved logarithmic spiral blades.

METHOD OF SOLUTION

A general analysis is developed in reference 1 for steady, two-dimensional, nonviscous, compressible flow in centrifugal compressors with arbitrary blade shapes and arbitrary variations in the passage height. The analysis is limited to radial- and conical-flow compressors in which the center line of the passage generates a right circular cone when rotated about the axis of the compressor (fig. 1). The two-dimensional flow pattern is considered to lie on the surface of this cone. The final equations, developed from the equations of absolute irrotational motion, continuity, and energy, in reference 1 are presented together with a brief discussion of the coordinate system, the assumptions and limitations of the general analysis, and the relaxation methods used in the numerical procedure.

Coordinate system. - A developed view of the conic surface generated by the passage center line (fig. 1) is shown in figure 2. The dimensionless conic coordinates of a fluid particle on the conic surface are R and θ . (All symbols are defined in the appendix.) The conic-radius ratio R is defined by

$$R = \frac{r}{r_T} \quad (1)$$

where r is the conic radius (distance along conic element from apex of cone) and the subscript T refers to the impeller tip. The coordinate system (R, θ) rotates with the angular velocity ω of the impeller. The passage-height ratio H in the direction normal to the conic surface (fig. 2) is a continuous function of the conic-radius ratio R .

$$H = \frac{h}{h_T} = f(R) \quad (2)$$

where h is the passage height at any conic-radius ratio R .

Assumptions and limitations. - This general analysis assumes that flow conditions are uniform across the passage normal to the conic surface; that is, the flow varies only along the conic surface. In order to satisfy this assumption, it is necessary that: (1) the gradient of h with respect to r be small; and (2) the cone angle α be sufficiently large. The allowable variation in α from 180° depends on the ratio h/r and on the desired accuracy. For the hypothetical limiting case in which h/r approaches zero everywhere along the conic surface, the analysis is accurate for all values of α .

Velocity-ratio components. - The fluid particle on the developed conic surface in figure 2 has a relative tangential-velocity ratio U and a radial- (along conic element) velocity ratio V . These velocity ratios are defined by

$$U = \frac{u}{c_o} \quad (3a)$$

$$V = \frac{v}{c_o} \quad (3b)$$

where

c local speed of sound

u tangential component of velocity relative to impeller (positive in direction of rotation)

v radial (along conic element) component of velocity

Subscript:

o absolute inlet stagnation condition

Stream function. - A dimensionless stream function ψ satisfies the continuity equation if defined as

$$\frac{\partial \psi}{\partial \theta} \equiv \frac{\rho}{\rho_o} VHR \quad (4a)$$

and

$$\frac{\partial \psi}{\partial R} \equiv \frac{-\rho}{\rho_o} UH \quad (4b)$$

where ρ is the weight density.

The stream function ψ is constant along blade surfaces. If ψ is assigned the value 0 along the driving face of the blade (the blade surface in the direction of rotation), then the value of ψ along the trailing face of the blade (the blade surface opposed to the direction of rotation) is given by (reference 1)

$$\psi_t = \varphi \sigma_T \quad (5)$$

where

$$\sigma = \theta_t - \theta_d \quad (6)$$

and where

$$\varphi = \frac{W}{\rho_o a_T c_o} \quad (7)$$

where

$$a_T = B \sigma_T r_T h_T \quad (8)$$

where

σ included passage angle between blade surfaces at constant radius
($\theta_t - \theta_d$)

φ compressor flow coefficient

W compressor flow rate

a flow area (normal to conic surface)

B number of passages (or blades)

Subscripts:

d driving face of blade (blade surface in direction of rotation)

t trailing face of blade (blade surface opposed to direction of rotation)

Differential equation for stream-function distribution. - The differential equation for the stream-function distribution in compressors with thin, logarithmic-spiral blades is given by equation (35) of reference 1 in terms of transformed coordinates ξ and η

$$\begin{aligned}
& \frac{2M_T \frac{\rho}{\rho_0}}{(\varphi \sec \beta)^2} \exp \left[\frac{(m+2)(\xi - \eta \tan \beta)}{\varphi \sec^2 \beta} \right] = \frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \eta^2} - \\
& \frac{\partial \psi}{\partial \xi} \frac{\partial}{\partial \xi} \left(\log_e \frac{\rho}{\rho_0} \right) - \frac{\partial \psi}{\partial \eta} \frac{\partial}{\partial \eta} \left(\log_e \frac{\rho}{\rho_0} \right) - \\
& \frac{m}{\varphi \sec^2 \beta} \left(\frac{\partial \psi}{\partial \xi} - \tan \beta \frac{\partial \psi}{\partial \eta} \right) \quad (9)
\end{aligned}$$

where the impeller-tip Mach number M_T is defined as

$$M_T = \frac{\omega r_T \sin \frac{\alpha}{2}}{c_0} \quad (10)$$

and where β is the blade curvature angle between a radius (conic element) and the tangent to the blade surface (fig. 3). The angle β is a constant for a given logarithmic-spiral blade. For positive values of β , the blades are curved forward (in the direction of rotation); for negative values of β , the blades are curved backward; for β equal to 0, the blades are straight and lie along R.

The transformed coordinates ξ and η , which are related to R and θ by (reference 1)

$$\xi = \varphi (\log_e R + \theta \tan \beta) \quad (11a)$$

$$\eta = \varphi (\theta - \tan \beta \log_e R) \quad (11b)$$

have been introduced because they result in straight, parallel blades in the transformed $\xi\eta$ -plane. Such a transformation is desirable because it simplifies the solution of the differential equation by relaxation methods. Lines of constant ξ and η in the physical $R\theta$ -plane are shown for a typical logarithmic-spiral blade in figure 4. These transformed coordinates, given by equation (11), are developed in reference 1 for the conditions that $\eta = 0$ along the driving face of the blade, $\eta = \psi_t = \varphi \sigma_T$ along the trailing face, and $\theta = 0$ for $\eta = 0$ at the impeller tip $R = 1.0$.

In equation (9), the blade-height ratio H is assumed, for convenience, to vary with the conic-radius ratio according to

$$H = R^m \quad (12)$$

where m is an arbitrary exponent. (For $m = 0$ the blade height remains constant and for $m = -1.0$ the flow area remains constant.)

In order to solve equation (9), it is necessary to know the density ratio, which is related to the impeller-tip Mach number and the relative velocity ratio Q by (equation (11), reference 1)

$$\frac{\rho}{\rho_o} = \left\{ 1 + \frac{\gamma-1}{2} \left[(RM_T)^2 - Q^2 \right] \right\}^{\frac{1}{\gamma-1}} \quad (13)$$

where γ is the ratio of specific heats and

$$Q^2 = U^2 + V^2 \quad (14)$$

and where the absolute whirl of the fluid ahead of the impeller is assumed to be zero. The velocity Q (multiplied by ρ/ρ_o) is in turn given by equations (4), (11), (12), and (14)

$$\frac{\rho}{\rho_o} Q = \frac{\varphi \sec \beta}{\exp \left[\frac{(m+1)(\xi - \eta \tan \beta)}{\varphi \sec^2 \beta} \right]} \left[\left(\frac{\partial \psi}{\partial \xi} \right)^2 + \left(\frac{\partial \psi}{\partial \eta} \right)^2 \right]^{\frac{1}{2}} \quad (15)$$

Equations (9), (13), and (15) provide three equations with three unknowns, ρ/ρ_o , Q , and ψ .

Numerical procedure. - The system of equations (9), (13), and (15) is solved by relaxation methods to obtain the stream-function distribution within the compressor. From this distribution, the velocity components and other conditions can be determined using equations (4), and so forth. Detailed outlines for the numerical procedure are given in references 1 to 3. The procedure is briefly sketched herein.

(1) Equations (9) and (15) are changed to finite-difference form (reference 2).

(2) Values of ψ are specified on the boundaries of the flow region. Because the flow region investigated by the solutions in this report does not extend as far upstream as the impeller inlet, the boundary values of ψ across the passage between blades at the upstream boundary are determined by the simplified analysis subsequently given. Other boundary values of ψ are determined by methods similar to those in references 1 and 2.

(3) Values of ψ are estimated at equally spaced points of a grid system within the boundaries of the flow region.

(4) The preceding estimated values of ψ are adjusted (relaxed) by the relaxation process until they satisfy equation (9) in finite-difference form.

(5) The downstream boundary values of ψ in the vaneless diffuser are adjusted to satisfy the Joukowski condition for tangency of flow leaving the tip of the impeller blade.

(6) After the Joukowski condition has been satisfied, the grid spacing is reduced near the impeller tip in order to obtain detailed knowledge of the flow characteristics in this region where conditions are rapidly changing.

RELAXATION SOLUTIONS

Numerical Examples

Two numerical examples are presented for two-dimensional, compressible flow through impellers with backward-curved, logarithmic-spiral blades with the blade angles β (fig. 3) equal to $\tan^{-1}(-0.5)$ and $\tan^{-1}(-1.0)$. A third example, for straight blades $\beta = \tan^{-1}(0)$, is also given. Logarithmic-spiral blades were selected because the known transformation of coordinates given by equation (11) for this particular blade shape considerably reduced the labor that would have been involved in the relaxation solution of an arbitrary blade shape. It is considered that the results of these solutions can be applied in a qualitative manner to blades with arbitrary curvature. The other design and operating parameters, which are the same for all three examples, are as follows:

Flow coefficient, ϕ	0.5
Impeller-tip Mach number, M_T	1.5
Exponent for variation in H with R , m	-1.0
Ratio of specific heats, γ	1.4
Included passage angle between blade surfaces at a constant radius, σ , deg	18

(For infinitely thin blades, σ is independent of R so that
 $\sigma_T = \sigma = \text{constant}$.)

The solutions were obtained in a region of the compressors (including the impeller tip) that was considered to be unaffected by the impeller-inlet configuration and by the diffuser vanes; that is, the diffuser vanes (if any) and the impeller inlet must be far enough removed from the region investigated not to affect appreciably the flow in that region. Each solution applies, within the limitations imposed by the assumption of two-dimensional flow, to conical-flow compressors (and turbines) with various cone angles α (including the radial-flow compressor, $\alpha = 180^\circ$) but with the same included passage angle σ (reference 1).

Results

The numerical results are presented in plots of the streamlines, constant Mach number lines, and constant-pressure-ratio lines in figures 5 to 7. The value of ψ obtained by relaxation methods at each of the grid points in the flow field is given for the three numerical examples in tables I to III. The velocities, pressures, and so forth, can be obtained from these values of ψ with equations (4), (13), and (14) by methods outlined in reference 1. The information obtained from these tables could be used to develop correlation equations (similar to those in reference 2) for predicting flow conditions in the impeller at other design and operating conditions, although past attempts to develop such correlation equations for logarithmic-spiral blades have failed.

Streamlines. - The streamline configurations (relative to the impeller) for the three examples are shown in figure 5. In order to assist in the proper geometrical orientation of the curved-blade examples, a dashed outline of the straight-blade impeller is superimposed on the flow field of the curved-blade solutions in figures 5 to 7. The streamlines are designated in such a manner (ψ/ψ_t) that the value of a streamline indicates the percentage of the total flow (in the passage) between the streamline and the driving face of the blade.

For a given density ratio and blade height, the streamline spacing is indicative of the velocities relative to the impeller, with close spacing indicating high velocities and wide spacing indicating low velocities.

The streamlines for the straight-blade example are given in figure 5(a). For the design and operating conditions of this example, an eddy has formed on the driving face of the blade. This eddy indicates negative velocities on and near the blade face and results from the condition of absolute irrotational motion. The eddy is attached to the blade and rotates with an angular velocity equal and opposite to the rotational velocity of the impeller. If the flow rate through the impeller were 0, this eddy would occupy the entire passage. In actual practice this eddy is probably unstable and should be eliminated by proper changes in the design and operating conditions of the compressor. From a comparison of figures 5(a), 5(b), and 5(c), it appears that the eddy can be reduced or eliminated by backward-curved logarithmic-spiral blades (for the same impeller-tip speed, number of blades, and compressor flow rate).

Mach number lines. - Lines of constant Mach number relative to the impeller are shown for the three examples in figure 6. The general characteristics of these plots are similar. The velocities (as indicated by the Mach number lines) along the driving face of the blade are low; the velocities along the trailing face are high; and the velocities become equal on both faces at the blade tip (as required by the Joukowski condition). The maximum relative Mach number occurs on the trailing face of the blade at a radius ratio well within the impeller and the flow decelerates along the face of the blade from this point to the blade tip. This deceleration is seen to decrease, however, as the blades become more backwardly curved.

If boundary-layer effects are neglected, the nonuniformity of the velocity at the impeller tip becomes negligible at a radius ratio of approximately 1.15. The average relative Mach number at the impeller tip is low because of the high impeller-tip Mach number, which results in high fluid densities and therefore low velocities.

Pressure-ratio lines. - Lines of constant static-pressure ratio (local pressure divided by the absolute inlet stagnation pressure) are shown for the three examples in figure 7. The general characteristics of these plots are the same. At a given radius ratio, the pressure ratio is higher on the driving face of the blade than on the trailing face, except at the blade tip where the pressure ratios are equal. This difference in pressure ratio accounts for the impeller torque.

It is interesting to note that the static-pressure ratios are essentially equal at the impeller tip in all three examples, although the total pressure must be progressively less as the blades become more backwardly curved. At the impeller tip, the absolute kinetic energy of the fluid (which is measured by the difference in total and static pressures) therefore becomes progressively less as the blade becomes more backwardly curved. The diffuser problem should therefore be less critical for backward-curved blades than for straight blades.

Velocity ratios along blade surfaces. - Of special interest, because of boundary-layer considerations, are the relative velocity-ratio distributions along the trailing and driving faces of the impeller blades, Q_t and Q_d . These velocities are plotted in figure 8 as a function of the radius ratio R . Negative values of Q along the driving face of the blade indicate the presence of a relative eddy on the blade surface. The deceleration is seen to be less rapid on the trailing face of the blade near the tip when the blade is curved farther backward. The velocities along the trailing face of the blade away from the impeller tip ($R = 1.0$ at tip) are about equal for all three examples. This equality results because, although the difference in velocities on the two blade surfaces becomes less as the blades are curved farther backward, the mean relative velocity must increase to accomplish the required through flow, and as a result the net change in velocity on the trailing face with changes in β is negligible (for the range of β investigated in the three numerical examples).

A measure of the tendency for boundary-layer separation is given by the term

$$\frac{\delta}{Q} \frac{dQ}{ds}$$

where δ is the boundary-layer thickness and s is distance along the blade surface. In the case of laminar flow, for example, this term must not become more negative than a certain value (the magnitude of which is not discussed in this report) or separation will occur. In dimensionless form this term becomes

$$\frac{\delta/r_T}{Q} \frac{dQ}{dS}$$

where S is distance along the blade surface in units of the tip radius. For a given value of δ/r_T , the more negative the value of $\frac{1}{Q} \frac{dQ}{dS}$, the

more likely will boundary-layer separation occur. In figure 9, $\frac{1}{Q} \frac{dQ}{dS}$ is shown as a function of R at the critical region on the trailing face of the blade near the impeller tip for the three numerical examples. It is evident that backward-curved logarithmic-spiral blades are more desirable than straight blades with regard to velocity gradient on the trailing face of the blade (at least for the same impeller-tip speed, number of blades, and compressor flow rate). If it is desired to compare the velocity gradients for straight and logarithmic spiral blades at the same work input, that is, at the same value of μM_T (where the slip factor μ is defined as the ratio of the average absolute tangential velocity of the fluid leaving the impeller tip to the tip speed of the impeller), then

$$(\mu M_T^2)_{\text{straight}} = (\mu M_T^2)_{\text{log-spiral}} \quad (16)$$

For the numerical example with β equal to $\tan^{-1}(-1.0)$, the "equivalent" tip Mach number for a straight blade is, from equation (16),

$$M_T = \sqrt{\frac{0.7681}{0.8986}} \times 1.5 = 1.3868$$

where the numerical values for the slip factor are given in the next section. (Note that for straight blades the slip factor is independent of impeller-tip speed, reference 2.) With the use of the correlation equations of reference 2, the velocity gradients were determined for straight blades at the equivalent tip Mach number of 1.3868 and the result is given by the dashed curve in figure 9. For straight blades the velocity gradients were changed very little by the reduced equivalent tip Mach number so that the velocity gradients for logarithmic-spiral blades are definitely more desirable than those for straight blades at the same work input.

Slip factor. - A method of evaluating the impeller slip factor μ is given in appendix E of reference 1. The following values of μ were obtained for the three examples of this report:

Blade angle β	Slip factor μ
$\tan^{-1}(0)$	0.8986
$\tan^{-1}(-0.5)$	.8343
$\tan^{-1}(-1.0)$	.7681

As would be expected, the slip factor is less for backward-curved blades than for straight blades because for backward-curved blades the tangential component of the relative velocity is opposed to the direction of rotation. If the slip factor is corrected by adding this tangential component of the relative flow, assuming the fluid is perfectly guided, the corrected slip factors become

Blade angle β	Slip factor μ (corrected)
$\tan^{-1}(0)$	0.8986
$\tan^{-1}(-0.5)$	.9013
$\tan^{-1}(-1.0)$	.9036

These corrected values of μ are essentially equal and it is therefore concluded that the slip velocity, which is defined as the average absolute tangential velocity at the impeller tip minus the absolute tangential velocity with perfect guiding by the blades, that is, assuming an infinite number of blades, is the same for various negative values of blade angle β . This conclusion is not in agreement with the well-known Stodola correction in which the slip velocity varies directly with $\cos \beta$ (reference 4, p. 1259).

SIMPLIFIED ANALYSIS

The relaxation solutions presented herein require a large number of calculations. It would therefore be advantageous to have a quicker (although less accurate) means of estimating the flow conditions in impellers with logarithmic-spiral blades. In this section, a simplified method of analysis is developed whereby flow conditions within the impeller, except near the tip, can be estimated for radial- and conical-flow impellers with logarithmic-spiral blades. The simplified analysis is based on the assumption that for high-solidity blades, such as those in centrifugal compressors, the component of the relative velocity normal to the blade is 0 at all radii within the impeller; that is, the fluid is perfectly guided by the blades.

Velocity distribution. - In terms of U , V , and M_T , the condition of irrotational motion given by equation (5) of reference 1 becomes

$$-2M_T = \frac{U}{R} + \frac{\partial U}{\partial R} - \frac{1}{R} \frac{\partial V}{\partial \theta} \quad (17)$$

But, from the assumption of this simplified analysis,

$$U = Q \sin \beta$$

$$V = Q \cos \beta$$

so that equation (17) becomes

$$-2M_T = \sin \beta \frac{Q}{R} + \sin \beta \frac{\partial Q}{\partial R} - \frac{\cos \beta}{R} \frac{\partial Q}{\partial \theta} \quad (18)$$

In terms of the transformed coordinates ξ and η , given by equation (11) and shown as functions of R and θ in figure 4, equation (18) becomes

$$-2M_T \exp \left(\frac{\xi - \eta \tan \beta}{\varphi \sec^2 \beta} \right) = Q \sin \beta - \frac{\varphi}{\cos \beta} \frac{dQ}{d\eta}$$

which integrates to give

$$Q = \frac{-M_T}{\sin \beta} \exp \left(\frac{\xi - \eta \tan \beta}{\varphi \sec^2 \beta} \right) - \text{constant} \exp \left(\frac{\eta \tan \beta}{\varphi \sec^2 \beta} \right) \quad (19)$$

The constant of integration is determined from the condition that $Q = Q_d$ when $\eta = 0$ (fig. 4); equation (19) then becomes

$$Q = Q_d \exp \left(\frac{\eta \tan \beta}{\varphi \sec^2 \beta} \right) + \frac{2M_T}{\sin \beta} \exp \left(\frac{\xi}{\varphi \sec^2 \beta} \right) \sinh \left(\frac{\eta \tan \beta}{\varphi \sec^2 \beta} \right) \quad (20)$$

Equation (20) gives the variation in Q with η along a line of constant ξ . (See fig. 4.) The determination of Q_d from continuity considerations is given in the following section.

If $\beta \rightarrow 0$, then from equation (11) $\xi = \varphi \log_e R$ and $\eta = \varphi \theta$; therefore, after the indeterminates are evaluated, equation (20) becomes

$$Q = Q_d + 2RM_T \theta \quad (20a)$$

which agrees with equation (40) of reference 1 for straight blades ($\beta = 0$).

The value of ξ in equation (20) can be expressed in terms of the radius ratio R_d at which ξ cuts the driving face of the blade ($\eta = 0$). From equation (11) with $R = R_d$ and $\eta = 0$,

$$\xi = \varphi(\log_e R_d + \theta \tan \beta)$$

$$0 = \varphi(\theta - \tan \beta \log_e R_d)$$

so that, after eliminating θ ,

$$R_d = \exp \left(\frac{\xi}{\varphi \sec^2 \beta} \right) \quad (21)$$

which can be substituted into equation (20).

Stream function. - The total differential for the stream function is given by

$$d\psi = \frac{\partial \psi}{\partial \theta} d\theta + \frac{\partial \psi}{\partial R} dR$$

which, from equation (4), becomes

$$\begin{aligned} d\psi &= \frac{\rho}{\rho_o} VHR d\theta - \frac{\rho}{\rho_o} UH dR \\ &= \frac{\rho}{\rho_o} HRQ \left(\cos \beta d\theta - \sin \beta \frac{dR}{R} \right) \end{aligned} \quad (22)$$

But, from equation (11),

$$d\eta = \varphi \left(d\theta - \tan \beta \frac{dR}{R} \right)$$

so that equation (22) becomes

$$d\psi = \frac{\rho}{\rho_0} HRQ \cos \beta \frac{d\eta}{\phi} \quad (23)$$

where ρ/ρ_0 is given by equation (13).

In order to determine Q_d in equation (20), equation (23) is numerically integrated across the passage to give

$$\psi_t = \frac{\cos \beta}{\phi} \int_0^{\eta_t = \psi_t = \phi \sigma_T} \frac{\rho}{\rho_0} HRQ d\eta \quad (24)$$

which equation is satisfied when the proper value of Q_d is contained in the expression for Q (equation (20)). After the value of Q_d is determined, the variation in ψ with η across the passage can be determined by the numerical solution of equation (23).

Comparison with relaxation solutions. - Comparisons between the results of the simplified analysis and the relaxation solutions are given in figures 10 to 12. In figure 10 the velocity ratio along the driving face of the blade Q_d is plotted as a function of the radius ratio along the driving face of the blade R_d for the three values of β : $\tan^{-1}(0)$, $\tan^{-1}(-0.5)$, and $\tan^{-1}(-1.0)$. The results of the simplified analysis are shown by the heavy curves; the relaxation solution is indicated by the plotted points. The agreement is considered to be good up to a conic radius ratio of 0.800 or 0.825. (This agreement exists for all values of $R < 0.800$ provided the impeller inlet can be neglected.)

In figure 11 the velocity ratio along the trailing face of the blade Q_t is plotted as a function of the radius ratio along the trailing face of the blade R_t for the three values of β used in the numerical examples. (Substantial agreement between the simplified analysis and the relaxation solution exists for all values of R less than those shown on the plot.) The value of R_t at which agreement fails, depends on the value of β . For β equal to 0, the agreement is satisfactory up to R_t equal to 0.800 or 0.825; for β equal to $\tan^{-1}(-0.5)$ the agreement is satisfactory up to R_t equal to approximately 0.9; for

β equal to $\tan^{-1}(-1.0)$ the agreement is satisfactory up to R_t equal to approximately 0.95. From equation (11), it can be shown that these values of R_t correspond to the values of R_d (for the same values of ξ) at which the agreement is satisfactory for Q_d along the driving face of the blade ($R_d = 0.800$ or 0.825 , fig. 10). The ratio of R_t/R_d for a given value of ξ is developed later and presented in equation (28). For σ_T equal to 18° and for the range of β investigated, the agreement between the relaxation solution and the simplified analysis therefore satisfactory for values of ξ corresponding to values of R_d up to 0.800 or 0.825.

In figure 12 the velocity ratio Q is plotted across the passage along lines of constant ξ for three values of β . The value of ξ for each value of β was selected to have a value of R_d between 0.800 and 0.825. The agreement between the relaxation solution and the simplified analysis is satisfactory.

Particular set of solutions. - In general, equation (24) must be integrated by numerical methods. If, however, the fluid is incompressible ($\rho/\rho_0 = 1$) and if the flow area is constant ($HR = 1$), equation (24) combined with equation (20) integrates across the passage from η and $\psi = 0$ on the driving face of one blade to η and $\psi = \psi_t$ on the trailing face of the next blade to give

$$\psi_t = \frac{2M_T}{\sin^2 \beta} \exp\left(\frac{\xi}{\phi \sec^2 \beta}\right) \left[\cosh\left(\frac{\psi_t \sin \beta}{\phi \sec \beta}\right) - 1 \right] + \frac{Q_d}{\sin \beta} \left[\exp\left(\frac{\psi_t \sin \beta}{\phi \sec \beta}\right) - 1 \right]$$

which, combined with equations (5) and (21), becomes

$$\frac{Q_d}{M_T} = \frac{\sin \beta}{\exp\left(\frac{\sigma_T \sin \beta}{\sec \beta} - 1\right)} \left\{ \sigma_T \frac{\phi}{M_T} - \frac{2R_d}{\sin^2 \beta} \left[\cosh\left(\frac{\sigma_T \sin \beta}{\sec \beta}\right) - 1 \right] \right\} \quad (25)$$

Equation (25) gives the variation in Q_d/M_T with R_d , β , σ_T , and ϕ/M_T . For incompressible flow, the speed of sound contained in the

denominators of Q , M_T , and φ has no physical significance and may have any arbitrary value because these quantities appear as ratios in equation (25). The quantity Q_d/M_T is the ratio of the velocity along the driving face of the blade to the tip speed of the impeller and φ/M_T is proportional to the ratio of the flow rate through the impeller to the tip speed of the impeller. It will be noted in equation (25) that Q_d/M_T decreases linearly with increasing radius ratio R_d .

The ratio Q_t/M_T along the trailing face of the blade can be determined from equation (20) with $\eta = \varphi\sigma_T$ and with R_d given by equation (21).

$$\frac{Q_t}{M_T} = \frac{2R_d}{\sin \beta} \sinh \left(\frac{\sigma_T \sin \beta}{\sec \beta} \right) + \frac{Q_d}{M_T} \exp \left(\frac{\sigma_T \sin \beta}{\sec \beta} \right) \quad (26)$$

where Q_d/M_T is given by equation (25). The value of R_d contained in equations (25) and (26) is related to the value of R_t , at which Q_t/M_T occurs, as follows:

From equation (11) eliminating θ

$$R = \exp \left(\frac{\frac{1}{\varphi} - \eta \tan \beta}{\sec^2 \beta} \right) \quad (27)$$

From equation (21) and because $R = R_t$ when $\eta = \varphi\sigma_T$, equation (27) becomes

$$R_t = R_d \exp \left(\frac{-\sigma_T \sin \beta}{\sec \beta} \right) \quad (28)$$

Equation (28) relates R_t and R_d the ratio of which is seen to be constant for a given impeller with logarithmic-spiral blades. It will be noted from equation (26) and (28) that Q_t/M_T increases linearly with increasing values of R_t .

In order to evaluate in a qualitative manner the effect of β upon the values of Q_d and Q_t , the variations in Q_d/M_T and Q_t/M_T with β for various values of R are plotted in figure 13 from equations (25) and (26) for $\sigma_T = 0.31416$ (18°) and $\varphi/M_T = 0.250$. It will be noted that β has a pronounced effect upon both Q_d/M_T and Q_t/M_T for large absolute values of β . In particular the value of

Q_d/M_T increases at an increasingly greater rate as the absolute value of β increases from 0. Because the minimum value of Q_d/M_T occurs in the immediate neighborhood of $\beta = 0$, the tendency for an eddy to form on the driving blade surface is greatest for straight blades. Therefore, at the same impeller tip speed, number of blades, and compressor flow rate, forward-curved blades ($\beta > 0$), as well as backward-curved blades ($\beta < 0$), have less tendency than straight blades ($\beta = 0$) for the formation of undesirable relative eddy flows on the driving face. This fact might be unexpected because for forward-curved blades the loading is higher and therefore the velocity on the driving face might be expected to be lower. In order to accomplish the required through flow, however, the relative velocities must increase as the blades are curved forward. This fact explains the very pronounced effect of large absolute values of β upon Q_d/M_T and Q_t/M_T .

In figure 13 the variation (with β) in the relative velocity ratios at the driving and trailing faces of the blades Q_d and Q_t at a radius ratio of 0.75 (for example) agrees with the variation shown in figure 8. In particular, in both figures the value of Q_t is somewhat lower for $\beta = \tan^{-1}(-0.5)$ than for $\tan^{-1}(-1.0)$ or $\tan^{-1}(0)$.

SUMMARY OF RESULTS AND CONCLUSIONS

Two numerical examples are presented for two-dimensional, compressible, nonviscous flow in centrifugal compressors with backward-curved, logarithmic-spiral blades, the center lines of which (in the axial-radial plane) generate a right circular cone when rotated about the axis of the compressor. The two examples for spiral blades are for different blade angles and are compared with a third example for straight, radial blades. All other design and operating parameters are the same for the three examples. The solutions were obtained in a region of the compressors (including the impeller tip) that was considered to be unaffected by the inlet configuration of the impeller and by the diffuser vanes. (That is, the impeller inlet and the diffuser vanes, if any, were far enough removed from the region investigated not to appreciably affect the flow in that region.) Each solution applies to radial- and conical-flow compressors (and turbines with the rotation and flow direction reversed) with various cone angles but with the same included passage angle between blades. The numerical results are presented in tables of the stream-function distribution and in plots of the streamlines, constant Mach number lines, and constant-pressure-ratio lines.

In addition, a simplified analysis for logarithmic-spiral blades is presented that can be used to determine flow conditions within the impeller except near the impeller tip and impeller inlet. For the range of blade angle β investigated and for an included passage angle between blades of 18° , the agreement between the relaxation solution and the simplified analysis was satisfactory for values of transformed coordinate ξ corresponding to values of conic-radius ratio along the driving face of blade up to 0.800 or 0.825. For a backward-curved blade of -45° and an included passage angle between blades of 18° the corresponding value of conic-radius ratio along the trailing face of blade at which agreement was good is as high as 0.95.

The principal conclusions of the work presented herein are:

1. The maximum relative Mach number occurs on the trailing face of the blade at a radius ratio well within the impeller and the flow decelerates along the face of the blade from this point to the blade tip. For the same impeller-tip speed, and so forth, this deceleration becomes less as the blades become more backwardly curved. Because of this lower rate of deceleration, backward-curved blades are more desirable than straight blades as far as boundary-layer separation is concerned.
2. The relative eddy, which appears on the driving face of the blade at low ratios of flow rate to impeller-tip speed, can be reduced or eliminated (at the same impeller-tip speed, and so forth) by backward-curved logarithmic-spiral blades. (Also see conclusion 5.)
3. The static-pressure ratios are essentially equal at the impeller tip for straight and backward-curved, logarithmic-spiral blades at the same impeller-tip speed. This fact indicates that at the impeller outlet the absolute kinetic energy of the fluid becomes progressively less as the blades become more backwardly curved. The diffuser problem should therefore be less critical for backward-curved blades.
4. The slip factor (defined as the ratio of the average absolute tangential velocity of the fluid leaving the impeller tip to the tip speed of the impeller) decreases as the blades are curved farther backward. If the slip factor is corrected by accounting for the tangential component of the relative flow, however, assuming perfect guiding of the fluid by the blades, this corrected slip factor is essentially independent of the blade angle.
5. From the simplified analysis, it is concluded that (at the same impeller-tip speed, and so forth) forward-curved blades ($\beta > 0$),

as well as backward-curved blades ($\beta < 0$), have less tendency than straight blades ($\beta = 0$) for the formation of undesirable relative eddy flows on the driving face of the blade.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, August 7, 1950.

APPENDIX - SYMBOLS

The following symbols are used in this report:

- a flow area normal to conic surface
- B number of passages (or blades)
- c local speed of sound
- exp exponential, $[\exp(x) = e^x]$
- H passage-height ratio, h/h_T
- h passage height normal to conic surface
- M_T impeller-tip Mach number, $\frac{\omega r_T \sin \frac{\alpha}{2}}{c_o}$
- m passage-height exponent
- p absolute static pressure
- Q relative velocity ratio, $\sqrt{U^2 + V^2}$
- R conic-radius ratio, r/r_T
- r conic radius (distance along conic element from apex of cone)
- S dimensionless distance along blade surface, s/r_T
- s distance along blade surface
- U relative tangential-velocity ratio, u/c_o
- u relative tangential velocity
- V radial-velocity ratio, v/c_o
- v radial velocity along conic element
- W total compressor flow rate

- α cone angle (fig. 1)
- β blade angle (fig. 3)
- γ ratio of specific heats
- δ boundary-layer thickness
- η transformed coordinate, equation (11b)
- θ angle, radians unless otherwise specified
- μ impeller slip factor
- ξ transformed coordinate, equation (11a)
- ρ weight density of fluid
- σ included passage angle between blade surfaces at constant radius,
 $\theta_t - \theta_d$
- φ compressor flow coefficient, $\frac{W}{\rho_o a_{T_o}^c}$
- ψ compressible stream function
- ω impeller angular velocity

Subscripts:

- d driving face of blade (blade surface in direction of rotation)
- o absolute inlet stagnation condition
- T impeller tip
- t trailing face of blade (blade surface opposed to direction of rotation)

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TABLE I - DISTRIBUTION OF STREAM FUNCTION ψ IN $\xi\eta$ -PLANE FOR $\beta = \tan^{-1} (0)$
 [Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$). $\eta_t = \phi\sigma = 0.15708$; ξ and η are functions of R and θ given by equation (11).]



ξ/η_t	η/η_t													
	0	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	5/4	13/16
-1 1/4	0		4.07		13.05		26.78		45.08		67.62		94.08	
-1 1/8	0		2.75		10.81		24.03		42.18		64.95		91.98	
-1	0		1.44		8.60		21.30		39.30		62.28		89.87	
-7/8	0		.16		6.46		18.68		36.52		59.68		87.78	
-3/4	0		-1.01		4.53		16.32		34.02		57.30		85.84	
-5/8	0		-1.98		3.01		14.50		32.08		55.41		84.22	
-1/2	0		-2.52		2.28		13.69		31.19		54.45		83.26	
-3/8	0		-2.31		2.89		14.57		32.08		55.09		83.49	
-1/4	0	-1.31	- .78	1.66	5.76	11.30	18.22	26.42	35.87	46.53	58.41	71.49	85.82	101.43
-3/16	0	- .65	.71	3.80	8.40	14.33	21.49	29.82	39.27	49.82	61.46	74.21	88.09	103.16
-1/8	0	.53	2.99	6.97	12.23	18.64	26.11	34.59	44.06	54.50	65.89	78.26	91.61	106.01
-1/16	0	2.45	6.42	11.51	17.56	24.52	32.34	41.00	50.50	60.82	71.95	85.90	96.65	110.23
0	0	5.63	11.52	17.86	24.77	32.30	40.49	49.34	58.87	69.08	79.95	91.47	103.60	116.30
1/16	3.07	11.50	19.08	26.56	34.25	42.33	50.86	59.90	69.47	79.58	90.21	101.33	112.88	124.79
1/8	12.02	21.25	29.78	38.07	46.39	54.92	63.76	72.97	82.58	92.60	103.02	113.80	124.88	136.17
3/16	25.37	34.88	43.91	52.71	61.48	70.37	79.45	88.80	98.45	108.41	118.66	129.17	139.89	150.73
1/4	42.71	52.28	61.56	70.68	79.75	88.90	98.18	107.66	117.34	127.28	137.37	147.69	158.13	168.66
5/8	88.25	107.59	126.56	145.58	164.95	184.76	204.94	225.24	245.49	266.37	286.31	306.11	326.36	346.85
1/2	149.11	168.63	187.96	207.29	226.78	246.49	266.37	286.31	306.11	326.36	346.85	367.31	387.80	408.25
5/8	225.80	245.39	264.90	284.40	303.97	323.63	343.36	363.11	382.80	402.55	422.30	442.05	461.80	481.55
3/4	319.10	338.73	358.35	377.98	397.60	417.23	436.85	456.48	476.10	495.73	515.36	535.00	554.63	574.26

TABLE II - DISTRIBUTION OF STREAM FUNCTION Ψ IN $\xi\eta$ -PLANE FOR $\beta = \tan^{-1}(-0.5)$

[Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$). $\eta_t = \phi\sigma = 0.15708$; ξ and η are functions of R and θ given by equation (11).]



ξ/η_t	η/η_t																
	0	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
-2 1/4	0		10.56		23.86		39.84		58.44		79.58		103.14		129.01		157.01
-2 1/8	0		9.77		22.50		38.15		56.55		77.91		101.83		128.26		157.01
-2	0		8.97		21.12		36.43		54.82		76.21		100.49		127.50		157.01
-1 7/8	0		8.14		19.69		34.64		52.93		74.46		99.10		126.70		157.01
-1 3/4	0		7.29		18.22		32.80		50.96		72.62		97.64		125.85		157.01
-1 5/8	0		6.40		16.70		30.89		48.92		70.70		96.10		124.95		157.01
-1 1/2	0		5.49		15.13		28.92		46.81		68.71		94.50		124.02		157.01
-1 3/8	0		4.57		13.55		26.93		44.67		66.68		92.86		123.05		157.01
-1 1/4	0		3.65		11.98		24.97		42.56		64.67		91.22		122.07		157.01
-1 1/8	0		2.78		10.51		23.14		40.59		62.78		89.65		121.11		157.01
-1	0		2.01		9.24		21.59		38.94		61.20		88.31		120.26		157.01
- 7/8	0		1.44		8.35		20.58		37.91		60.21		87.43		119.57		157.01
- 3/4	0		1.18		8.06		20.45		37.92		60.28		87.46	102.87	119.66	137.62	157.01
- 11/16	0		1.43		8.82		21.70		39.60	50.31	62.15	75.10	89.16	104.26	120.54	138.06	157.01
- 5/8	0								41.31	52.16	64.09	77.04	90.99	105.91	121.82	138.78	157.01
- 9/16	0		2.38		10.94	17.30	24.94	33.79	43.77	54.83	66.90	79.93	93.84	108.59	124.08	140.23	157.01
- 1/2	0				12.68	19.48	27.54	36.77	47.10	58.46	70.78	83.98	97.97	112.65	127.90	143.47	158.88
- 7/16	0	1.39	4.35	8.94	14.98	22.34	30.93	40.65	51.44	63.20	75.87	89.36	103.58	118.42	133.75	149.36	164.94
- 3/8	0	2.06	5.82	11.16	17.92	25.98	35.22	45.55	56.90	69.18	82.32	96.23	110.84	126.05	141.74	157.77	173.97
- 5/16	0	2.98	7.70	13.97	21.62	30.51	40.53	51.58	63.60	76.51	90.24	104.71	119.86	135.61	151.86	168.54	185.58
- 1/4	0	4.18	10.11	17.51	26.21	36.07	46.98	58.87	71.67	85.31	99.75	114.91	130.73	147.14	164.09	181.51	199.32
- 3/16	0	5.75	13.17	21.92	31.83	42.79	54.71	67.54	81.22	95.70	110.95	126.90	143.49	160.69	178.44	196.70	215.42
- 1/8	0	7.85	17.09	27.40	38.67	50.84	63.87	77.73	92.39	107.80	123.94	140.77	158.22				
- 1/16	0	10.81	22.20	34.22	46.95	60.42	74.63	89.59	105.29	121.70	138.80	156.57	174.97				
0		15.57	29.02	42.72	56.91	71.70	87.13	103.24	120.02		174.44						
1/16	1.87																
1/8	7.93	23.03	38.01	53.19	68.75	84.84	101.49	118.77	136.68								
3/16	16.96	33.10	49.32	65.80	82.66												
1/4	28.57	45.64	62.96	80.62													
3/8	58.41		97.05		137.45		136.32		176.21		218.53		263.34		310.65		360.43
1/2	97.44		140.40		185.42		179.87		224.53		271.61		321.18		373.28		427.92
5/8	145.69		193.17		242.93		232.62		282.16		334.14		388.66		445.78		505.44
3/4	203.42		255.64		310.29		295.06		349.53		406.50		466.13		528.53		593.47
7/8	270.91		328.08		387.75		367.37		426.98		489.26		554.22		621.76		692.01
1	348.43		410.78		475.71		450.03		514.97		582.55		652.76				
1 1/8	436.46		504.04		574.25		543.31		613.51								



TABLE III - DISTRIBUTION OF STREAM FUNCTION ψ IN η -PLANE FOR $\beta = \tan^{-1} (-1.0)$

Flow coefficient φ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$). $\eta_c = \varphi \sigma = 0.15708$; ξ and η are functions of R and θ given by equation (11).]

t/η_c	η/η_c																
	0	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
-2 1/4	0		10.68		23.96		39.75		58.23		79.19		102.66		128.56		157.00
-2 1/8	0		9.21		23.11		38.72		57.05		78.07		101.75		128.07		157.00
-2	0		9.72		22.26		37.65		55.88		76.95		100.85		127.55		157.00
-1 7/8	0		9.23		21.41		36.37		54.71		75.94		99.94		127.01		157.00
-1 3/4	0		8.75		20.58		35.32		53.57		74.75		99.05		126.48		157.00
-1 5/8	0		8.28		19.78		34.52		52.50		73.72		98.20		125.96		157.00
-1 1/2	0		7.86		19.08		33.65		51.56		72.82		97.44		125.49		157.00
-1 7/16	0				18.54		33.00		50.87		72.16		96.89	110.45	140.63	157.00	
-1 3/8	0		7.53						50.60		71.94		96.71	110.40	140.53	157.00	
-1 1/4	0		7.32		18.23		32.68					84.06	96.66	110.40	140.51	157.00	
-1 3/16	0		7.28		18.27		32.87		50.96		72.43	84.42	97.22	110.84	140.54	157.00	
-1 1/8	0									61.86	73.07	85.09	97.90	111.48	140.97	157.00	
-1 1/16	0		7.46		18.76		33.75		52.23	62.74	74.05	86.15	99.99	112.55	143.12	158.09	
- 15/16	0							43.49	53.32	63.98	75.44	87.66	100.60	114.22	128.43	145.85	161.07
- 7/8	0		7.94		19.86		35.54	44.73	54.77	65.63	77.29	89.70	102.82	116.60	130.97	145.85	161.07
- 13/16	0					28.30	36.85	46.29	56.59	67.71	79.63	92.31	105.69	119.75	134.41	149.63	165.34
- 3/4	0				21.70	29.61	38.46	48.21	58.82	70.26	82.50	95.51	109.24	123.66	138.72	154.40	170.66
- 11/16	0			15.64	22.93	31.19	40.39	50.50	61.47	73.29	85.90	99.32	113.46	128.32	143.86	160.06	176.91
- 5/8	0	4.83	10.87	17.96	24.39	33.05	42.65	53.17	64.57	76.83	89.90	103.75	118.36	133.72	149.80	166.58	184.05
- 9/16	0	5.33	11.83	19.41	28.04	37.67	48.26	59.77	72.18	85.45	99.56	114.48	130.20	146.70	163.96	182.03	200.83
- 7/16	0	5.88	12.93	21.07	30.27	40.47	51.64	63.73	76.71	90.56	105.26	120.78	137.13	154.27	172.21	190.94	220.79
- 3/8	0	6.50	14.17	22.95	32.79	43.62	55.42	68.14	81.74	96.23	111.56	127.72	144.73	162.55	181.18		
- 5/16	0	7.20	15.59	25.09	35.63	47.16	59.65	73.05	87.33	102.50	118.50	135.35	153.05	171.57			
- 1/4	0	8.01	17.21	27.51	38.83	51.12	64.34	78.47	93.47	109.35	126.08	143.65	162.06				
- 3/16	0	8.94	19.06	30.24	42.42	55.53	69.54	84.43	100.20	116.83	134.31	152.63					
- 1/8	0	10.04	21.20	33.36	46.45	60.43	75.28	90.98	107.55	124.96	143.22						
- 1/16	0	11.36	23.71	36.93	51.00	65.90	81.61	98.15	115.54	133.76							
0	1.09	15.70	30.58	45.98	62.02	78.77	96.26	114.53	143.73								
1/16	4.07	19.53	35.36	51.71	68.67	86.31	104.69										
1/8	8.34	24.49	41.13	58.53	76.16	94.66											
3/16	13.66	30.47	47.86	65.86	84.52												
1/4	19.91	37.39	55.51	74.28													
5/16	27.05	45.20	64.04		103.78												
3/8	35.03	53.88															
7/16	45.83																
1/2	63.79																
5/8	86.94																
3/4	113.26																
7/8	142.80																
1	175.84																
1 1/8	211.87																
1 3/8	251.59																
1 1/2																	

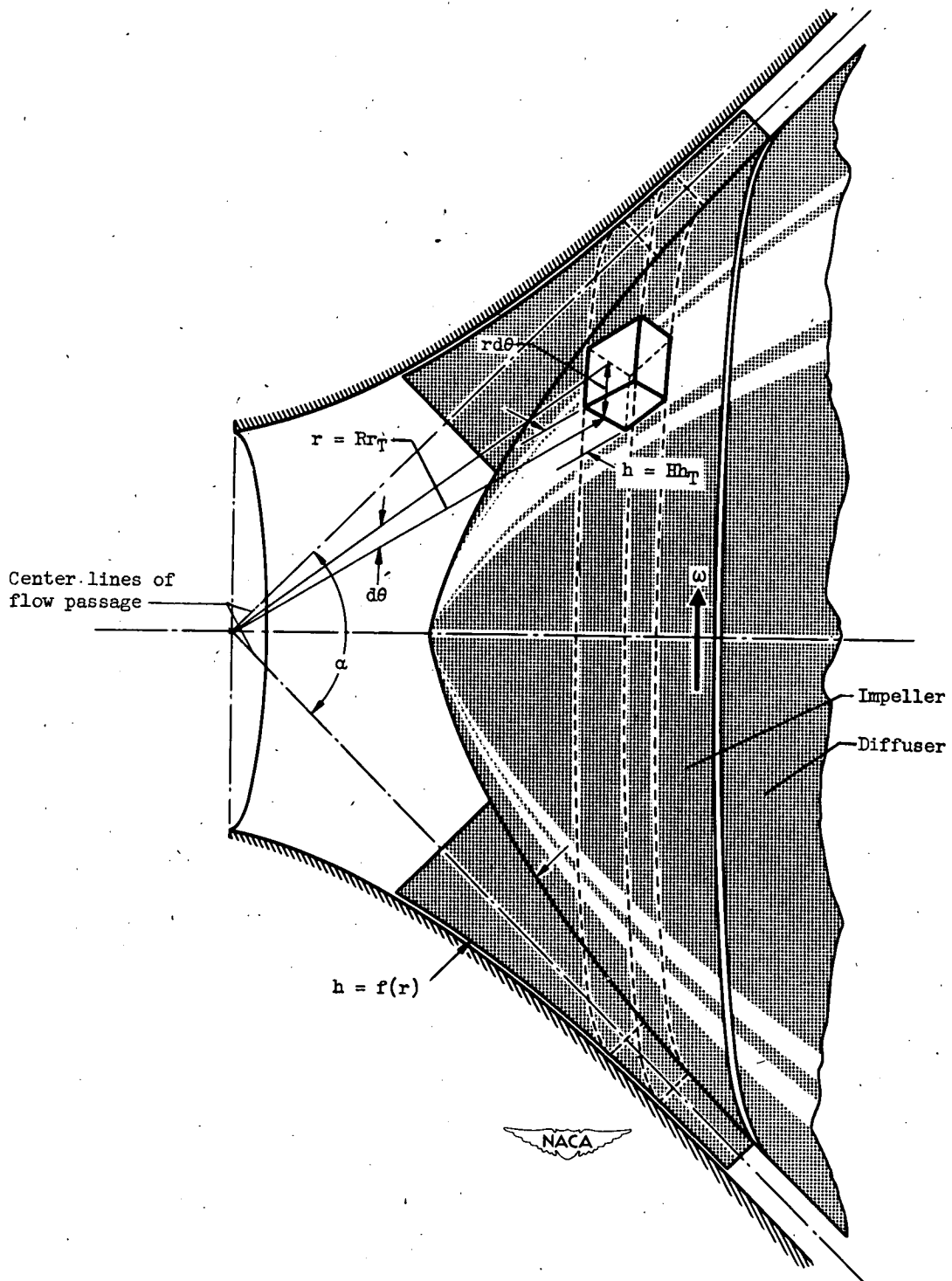


Figure 1. - Fluid particle on rotating coordinate system of impeller. Center line of flow passage generates right circular cone with cone angle α .

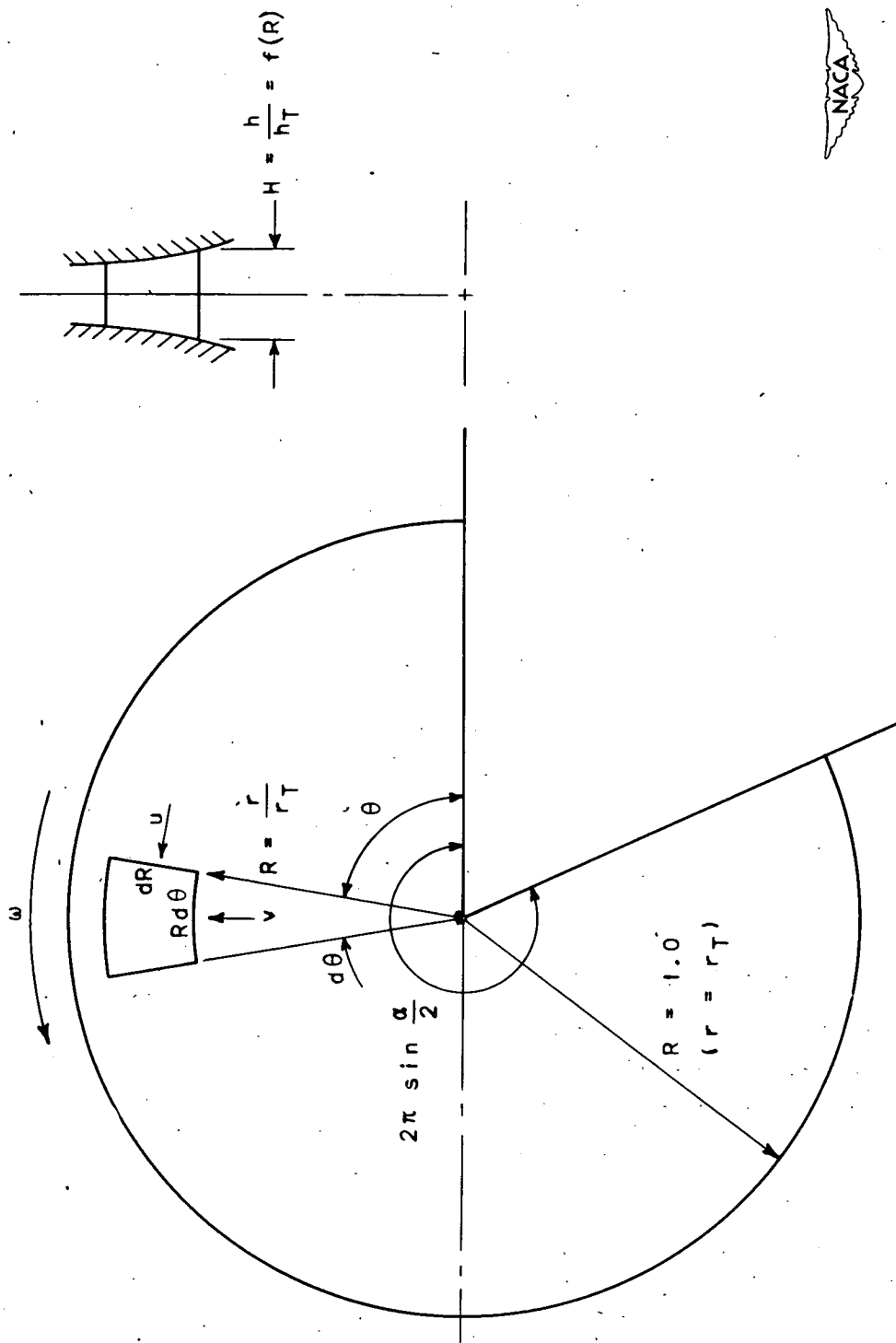


Figure 2. - Fluid particle on developed view of conic surface. R , θ , and H , dimensionless coordinates relative to impeller; u and v , tangential and radial components of velocity relative to impeller, respectively.

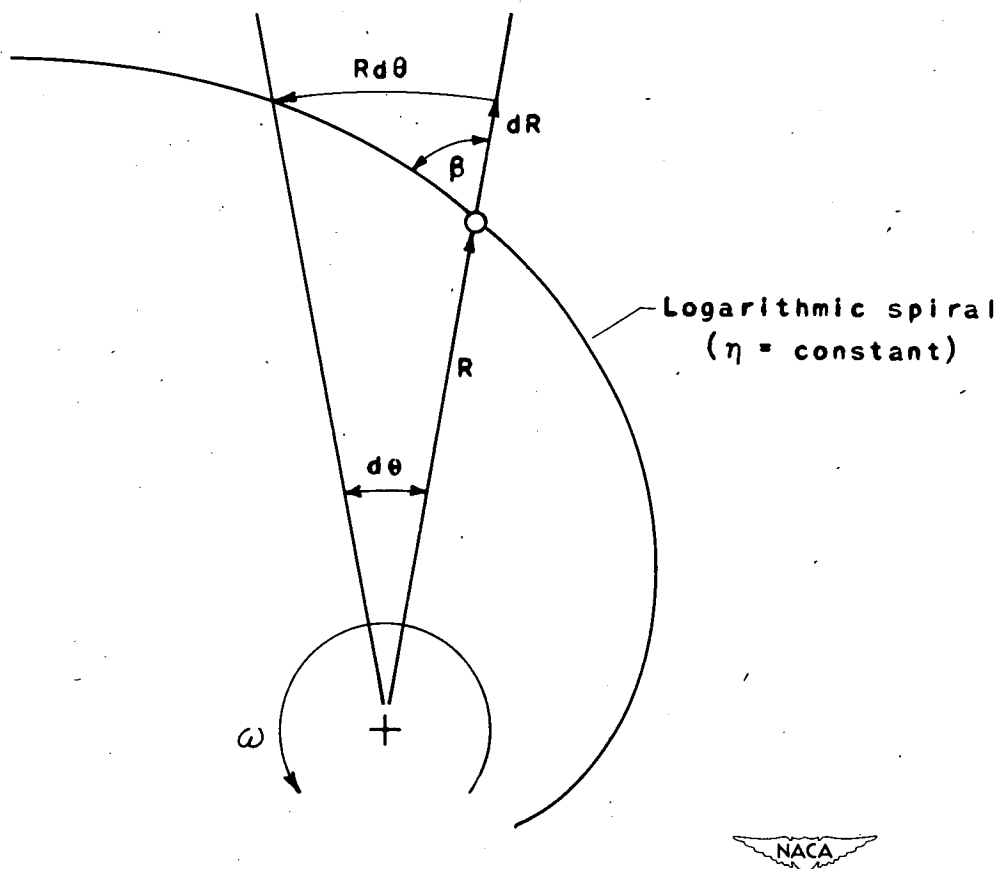


Figure 3. - Logarithmic-spiral blade shape.

Backward-curved blade, $\beta < 0$

Straight radial blade, $\beta = 0$

Forward-curved blade, $\beta > 0$

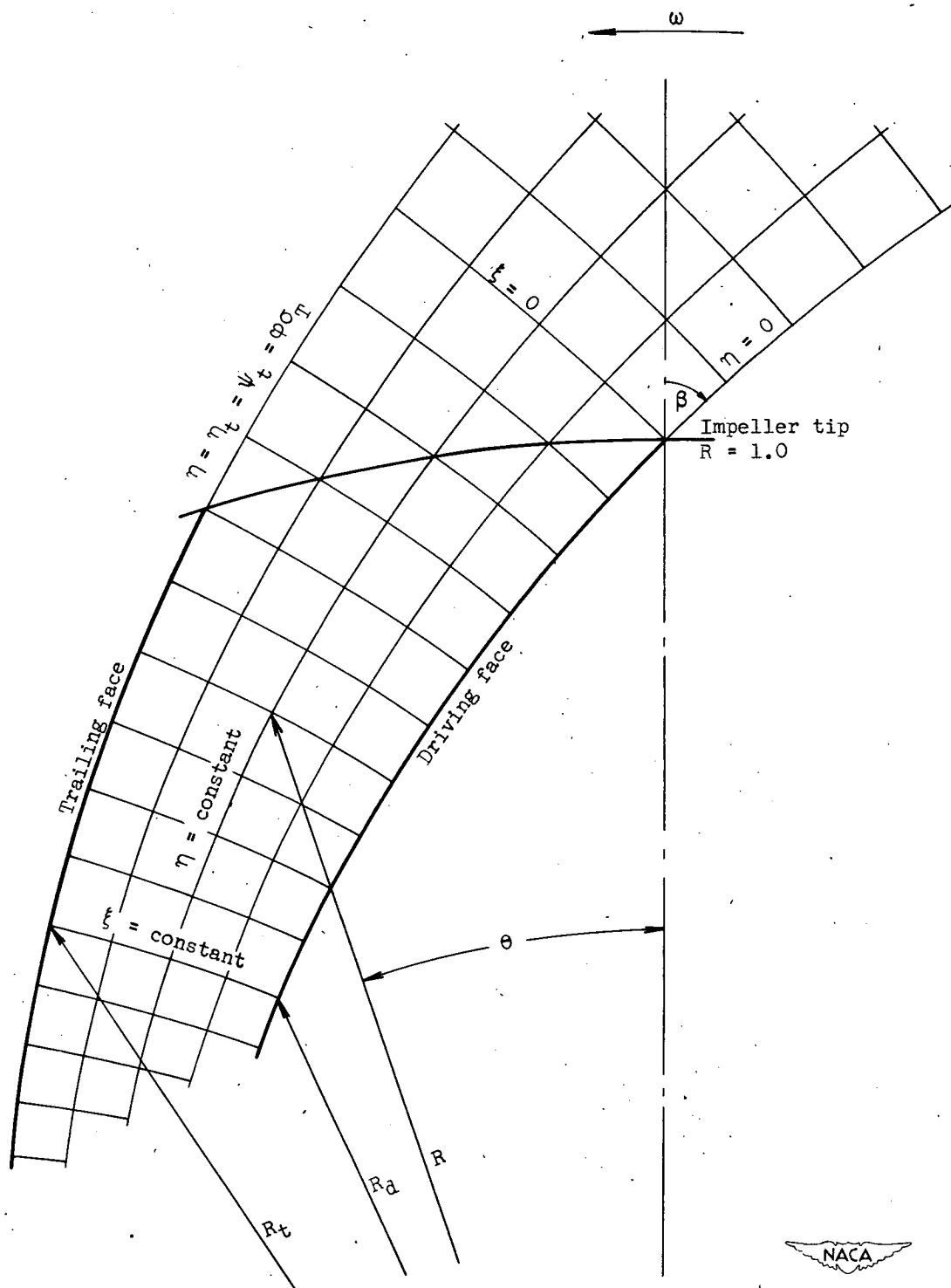
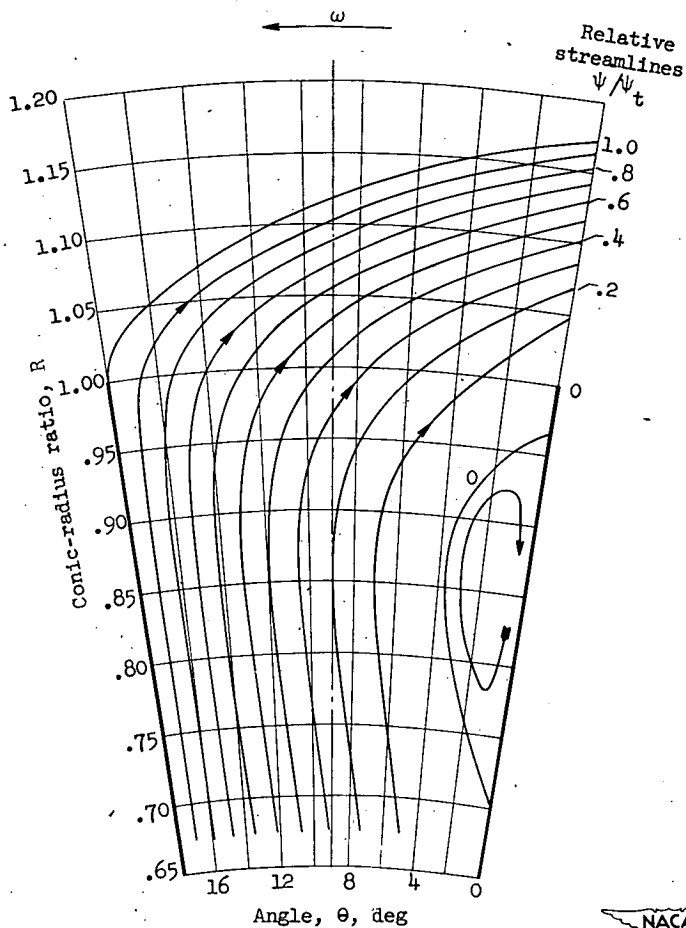
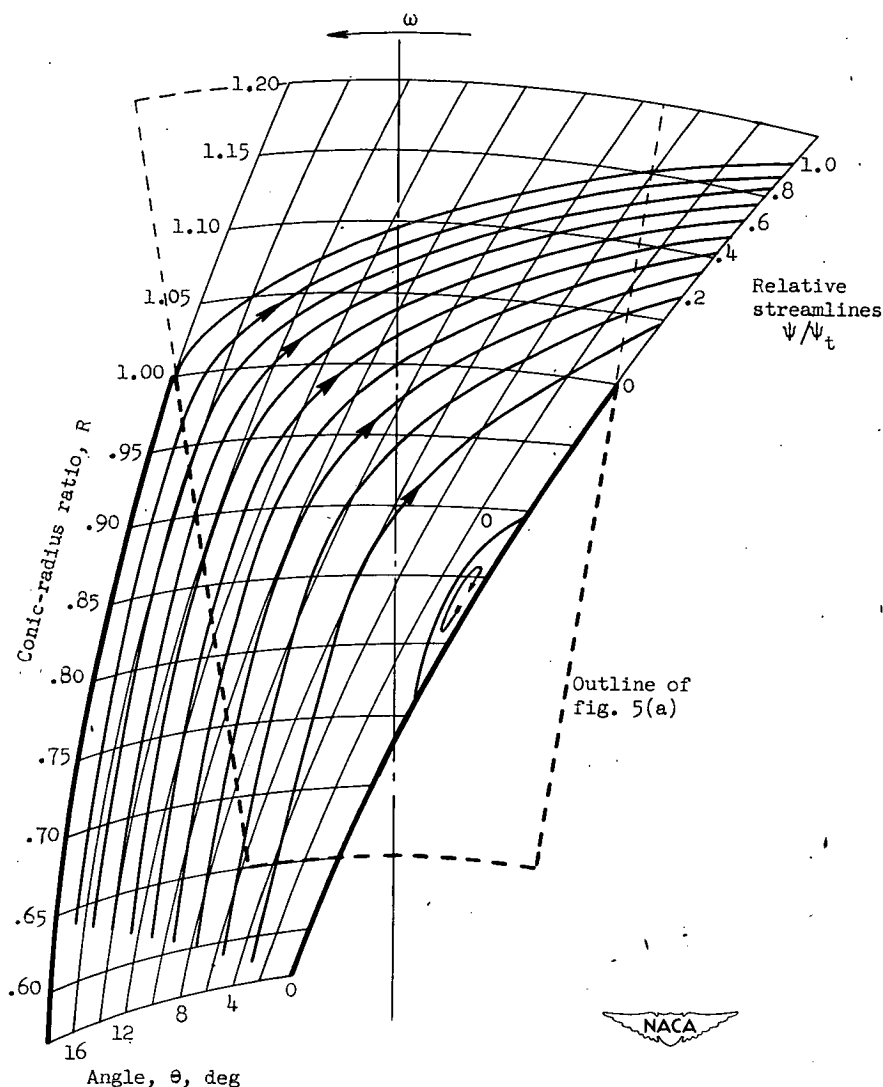


Figure 4. - Lines of constant ξ and η (transformed coordinates) for logarithmic-spiral blades in physical $R\theta$ -plane.



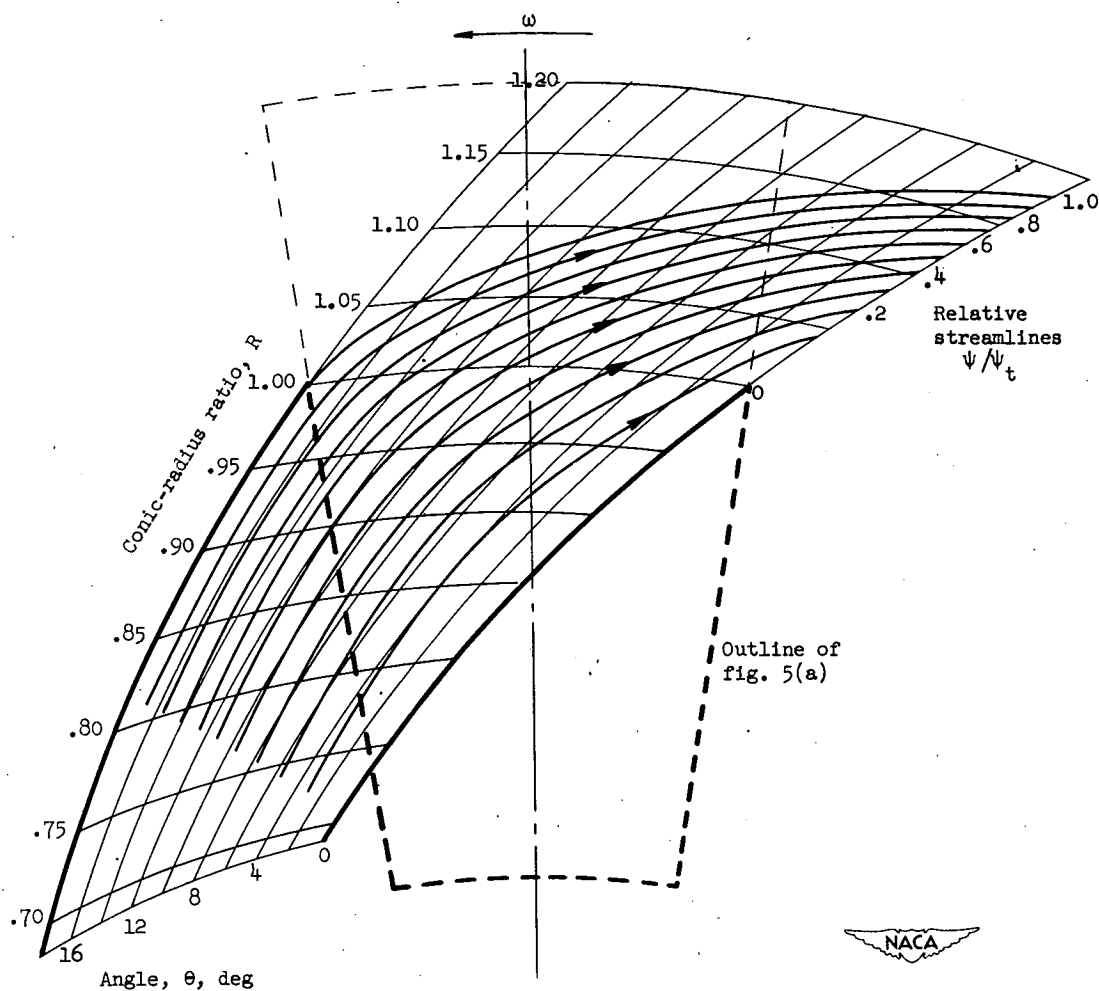
(a) Straight blades, $\beta = \tan^{-1}(0)$.

Figure 5. - Relative streamlines for compressible flow through centrifugal impellers with various values of blade angle β . Streamline designation ψ/ψ_t indicates percentage of total flow (through passage) between streamline and driving face of blade (right side of passage). Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).



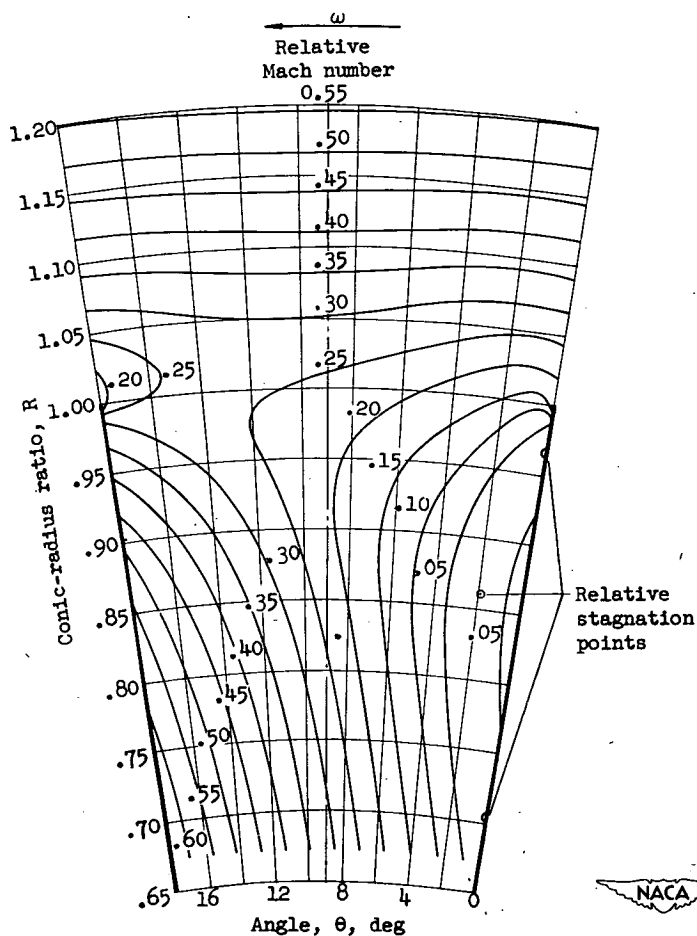
(b) Backward-curved blades, $\beta = \tan^{-1} (-0.5)$.

Figure 5. - Continued. Relative streamlines for compressible flow through centrifugal impellers with various values of the blade angle β . Streamline designation ψ/ψ_t indicates percentage of total flow (through passage) between streamline and driving face of blade (right side of passage). Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).



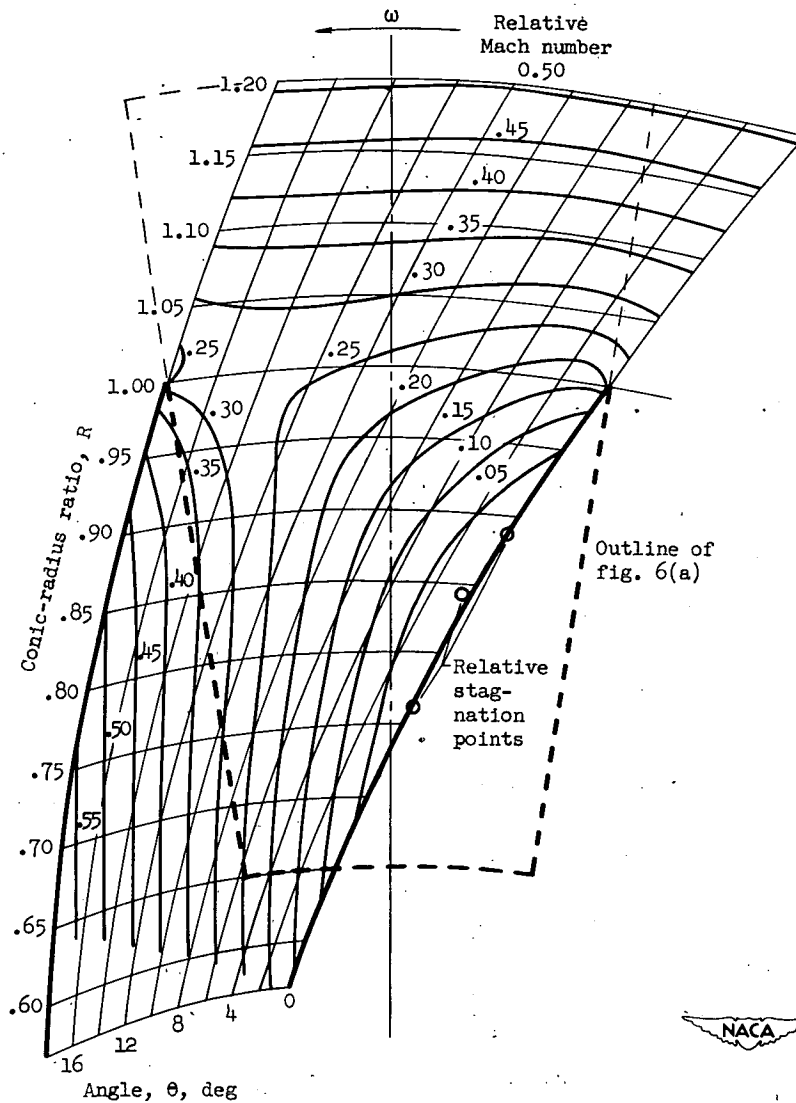
(c) Backward-curved blades, $\beta = \tan^{-1} (-1.0)$.

Figure 5. - Concluded. Relative streamlines for compressible flow through centrifugal impellers with various values of the blade angle β . Streamline designation ψ/ψ_t indicates percentage of total flow (through passage) between streamline and driving face of blade (right side of passage). Flow coefficient Φ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).



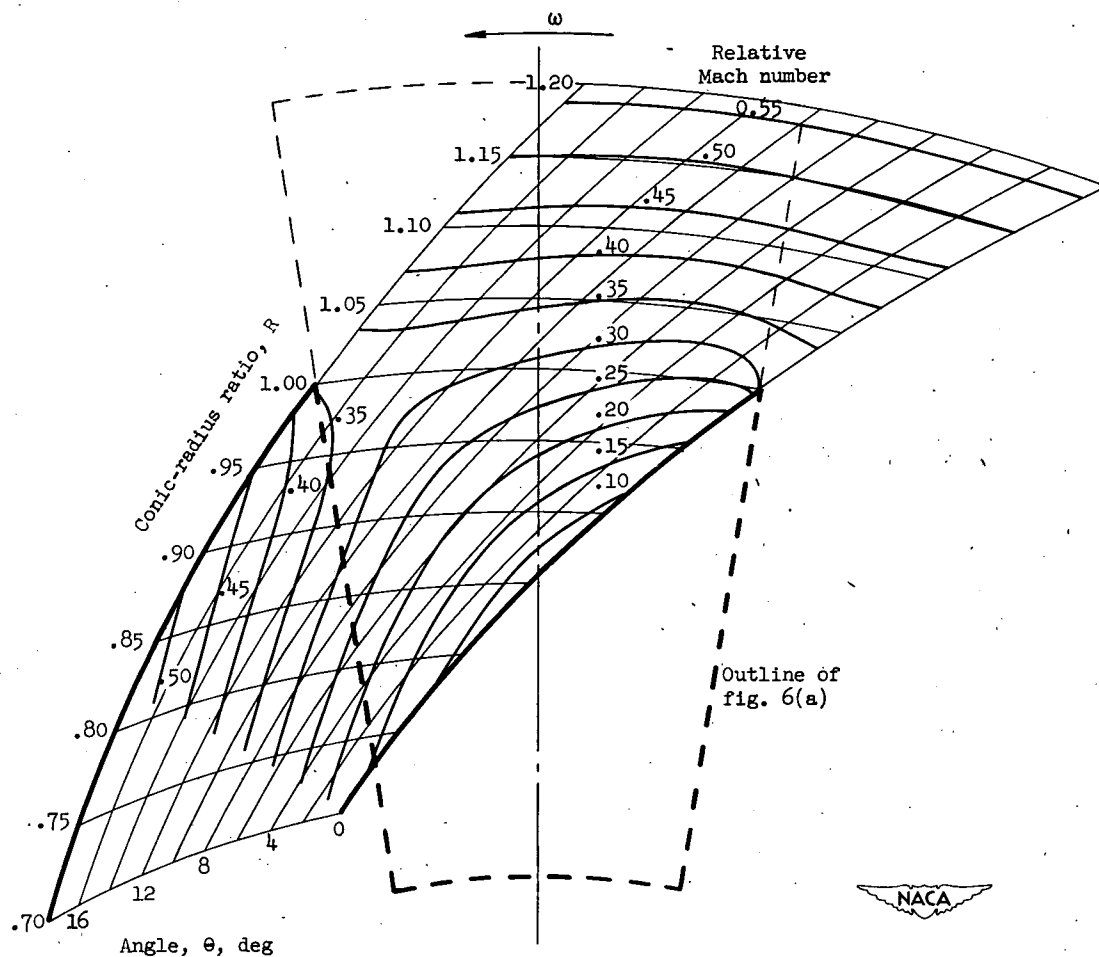
(a) Straight blades, $\beta = \tan^{-1}(0)$.

Figure 6. - Lines of constant Mach number relative to impeller. Flow coefficient Φ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 180° ; compressible flow ($\gamma = 1.4$).



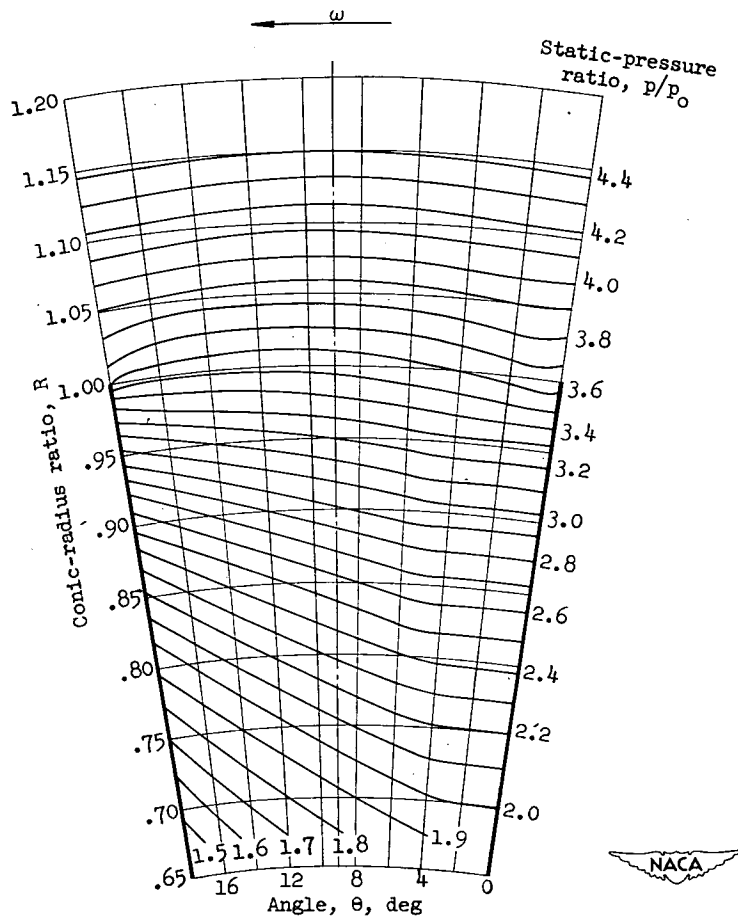
(b) Backward-curved blades, $\beta = \tan^{-1} (-0.5)$.

Figure 6. - Continued. Lines of constant Mach number relative to impeller. Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 180° ; compressible flow ($\gamma = 1.4$).



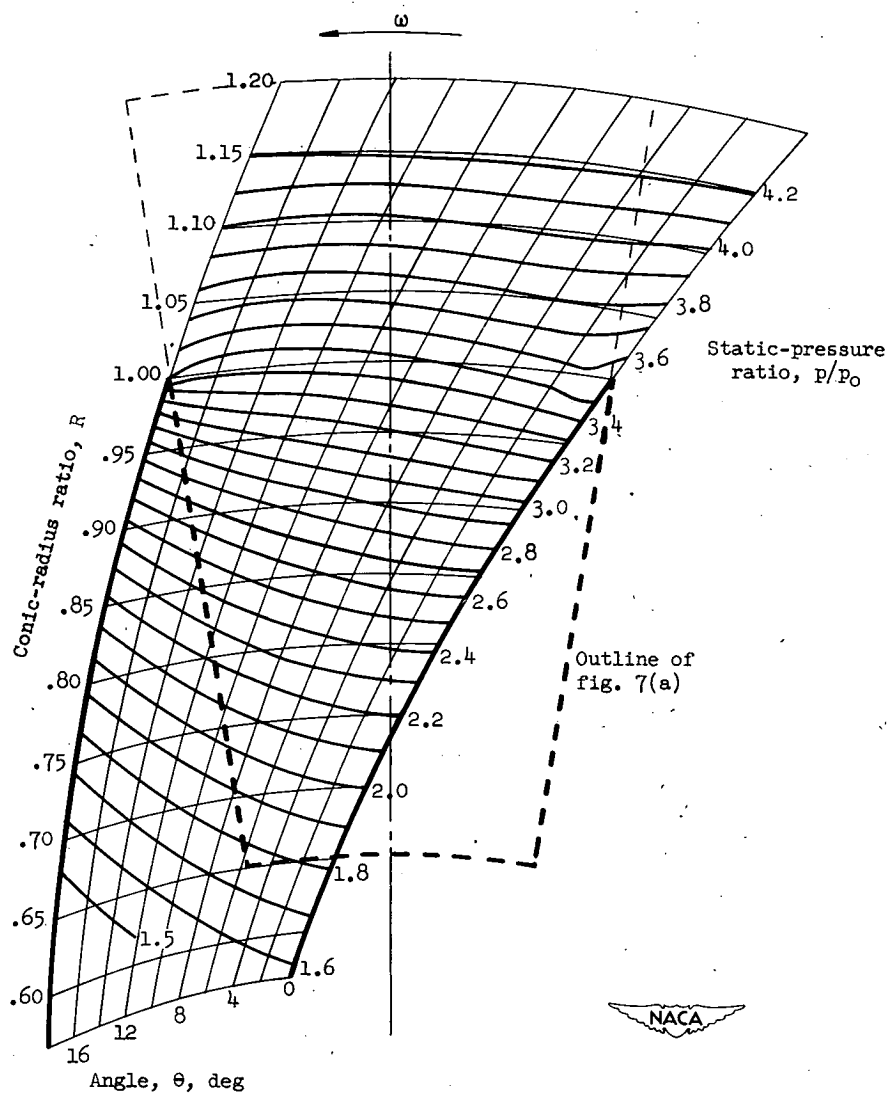
(c) Backward-curved blades, $\beta = \tan^{-1}(-1.0)$.

Figure 6. - Concluded. Lines of constant Mach number relative to impeller. Flow coefficient Φ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).



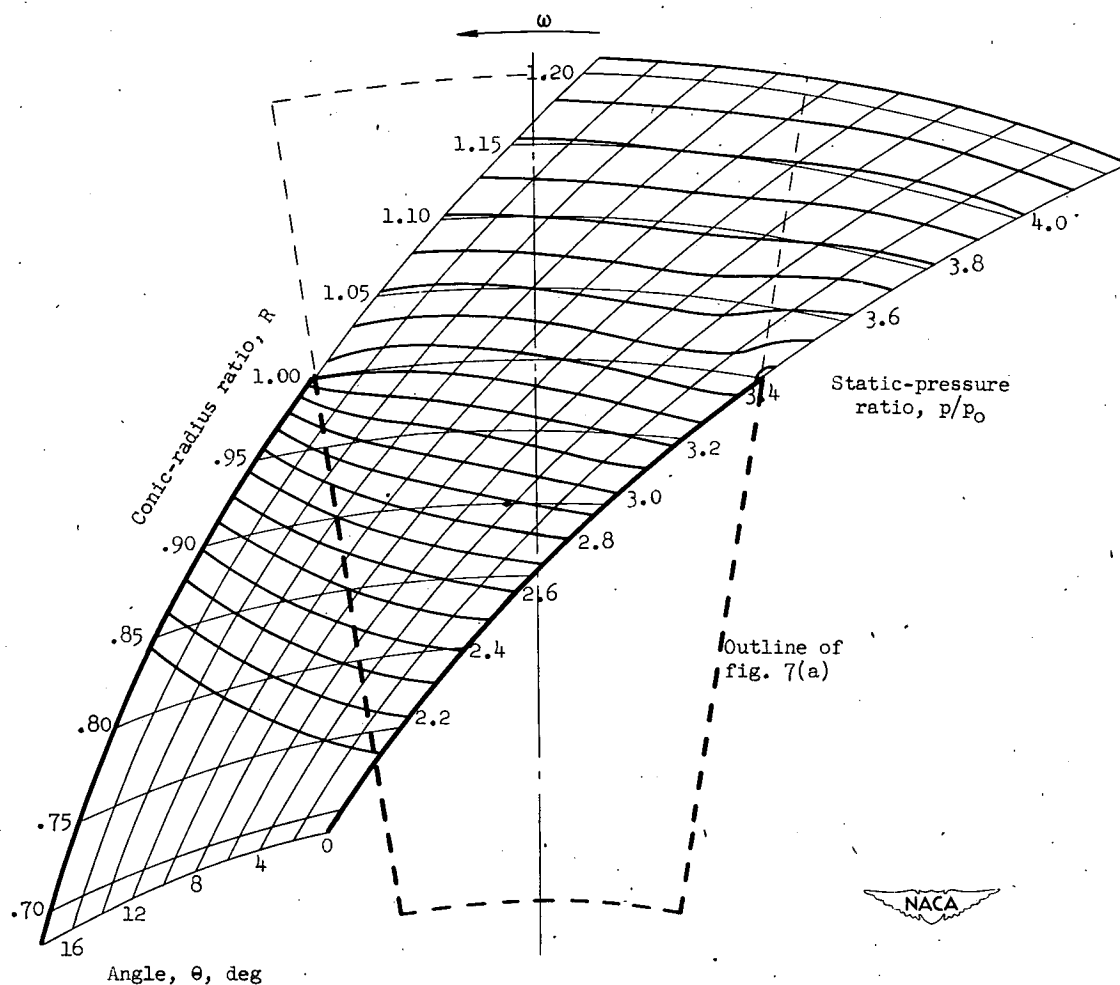
(a) Straight blades, $\beta = \tan^{-1}(0)$.

Figure 7. - Lines of constant static-pressure ratio. Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).



(b) Backward-curved blades, $\beta = \tan^{-1} (-0.5)$.

Figure 7. - Continued. Lines of constant static-pressure ratio. Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).



(c) Backward-curved blades, $\beta = \tan^{-1} (-1.0)$.

Figure 7. - Concluded. Lines of constant static-pressure ratio. Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).

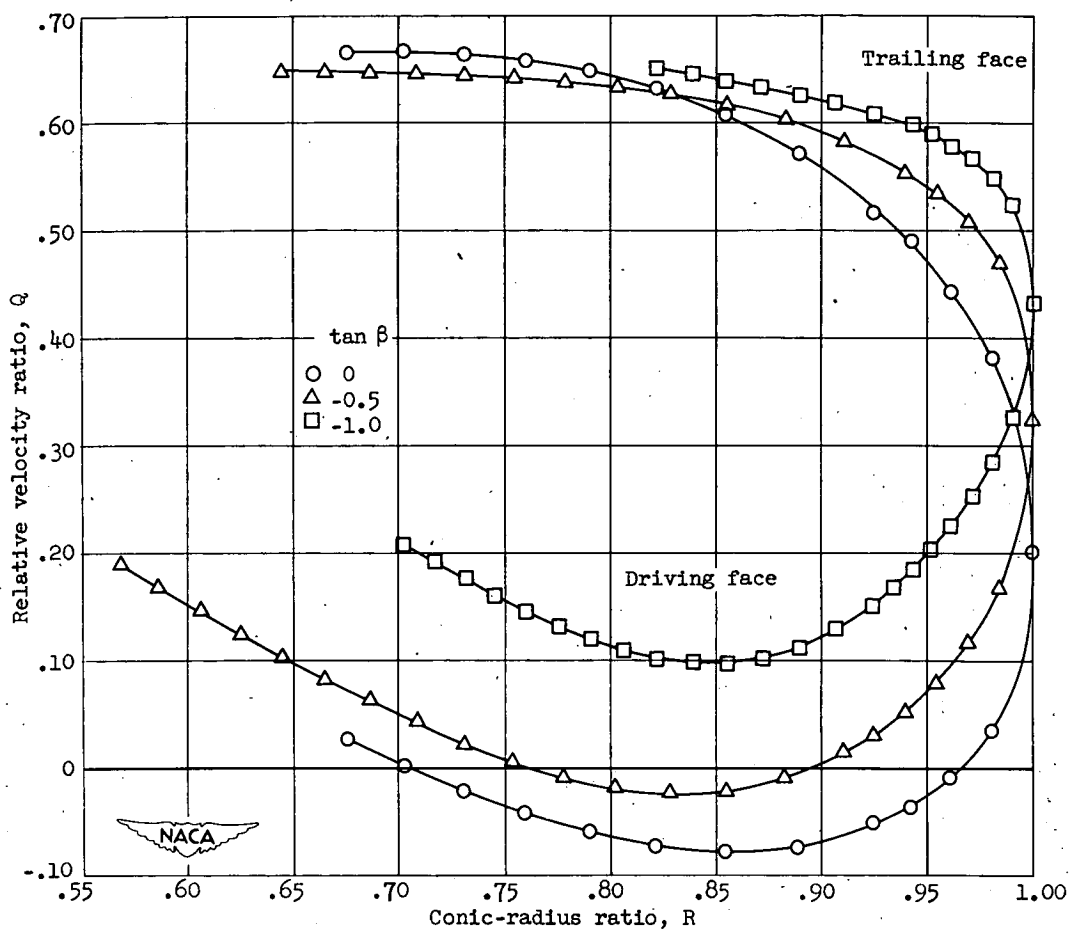


Figure 8. - Variation in relative ratio along driving and trailing faces of logarithmic-spiral blades for three values of blade angle β . Flow coefficient ϕ , 0.5; impeller-tip Mach number M_t , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).

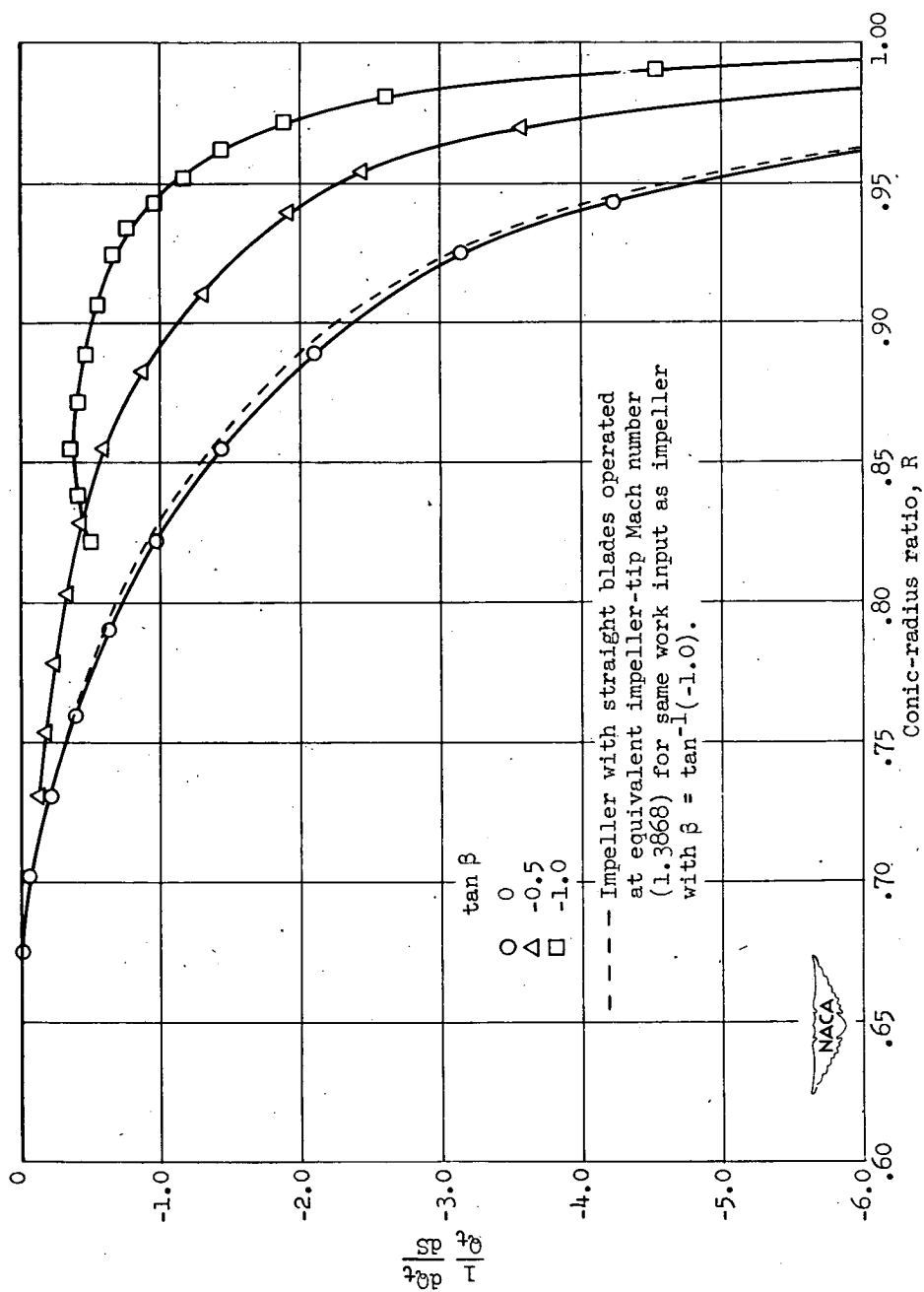


Figure 9. - Variation in $\frac{1}{Q} \frac{dQ}{dS}$ with conic-radius ratio along trailing face of blade for three values of blade angle β . Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades C , 18° ; compressible flow ($\gamma = 1.4$).

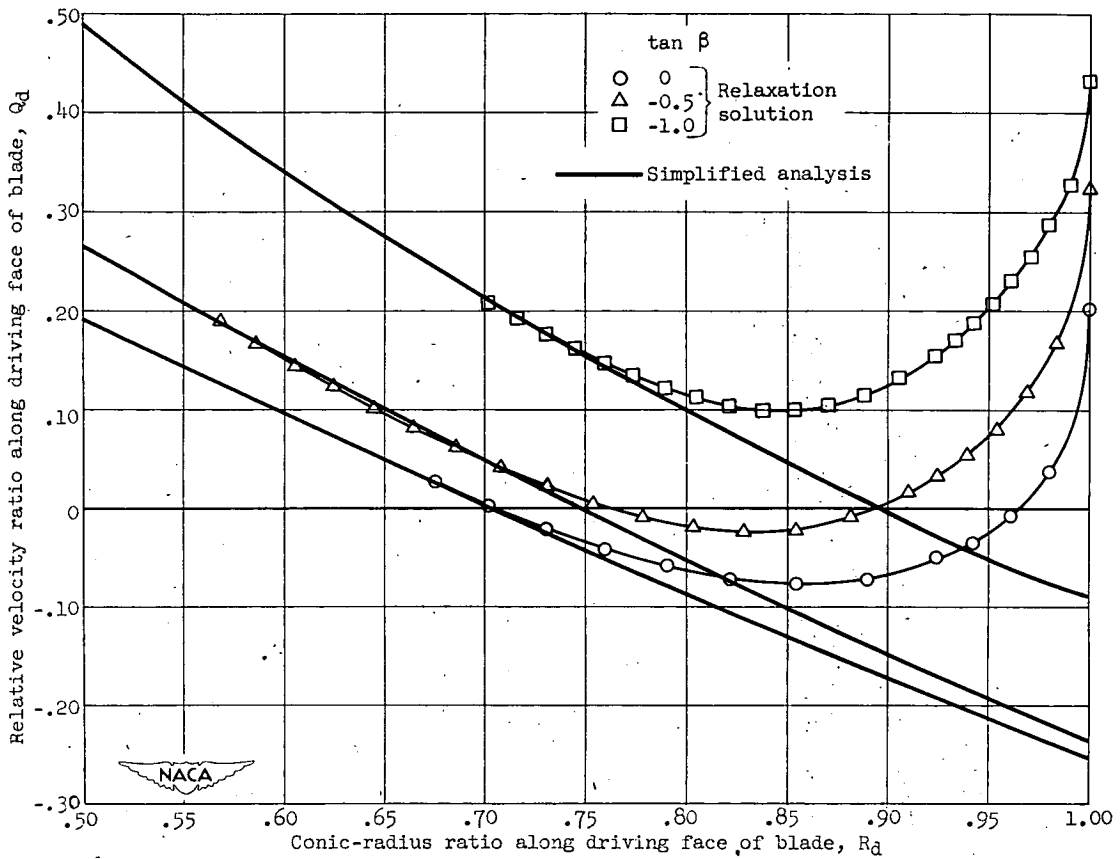


Figure 10. - Comparison of variation in relative velocity ratio along driving face of blade with conic-radius ratio along driving face of blade as obtained by relaxation methods and by the simplified analysis for three values of blade angle β . Flow coefficient ϕ , 0.5; impeller-tip Mach number M_t , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 180° ; compressible flow ($\gamma = 1.4$).

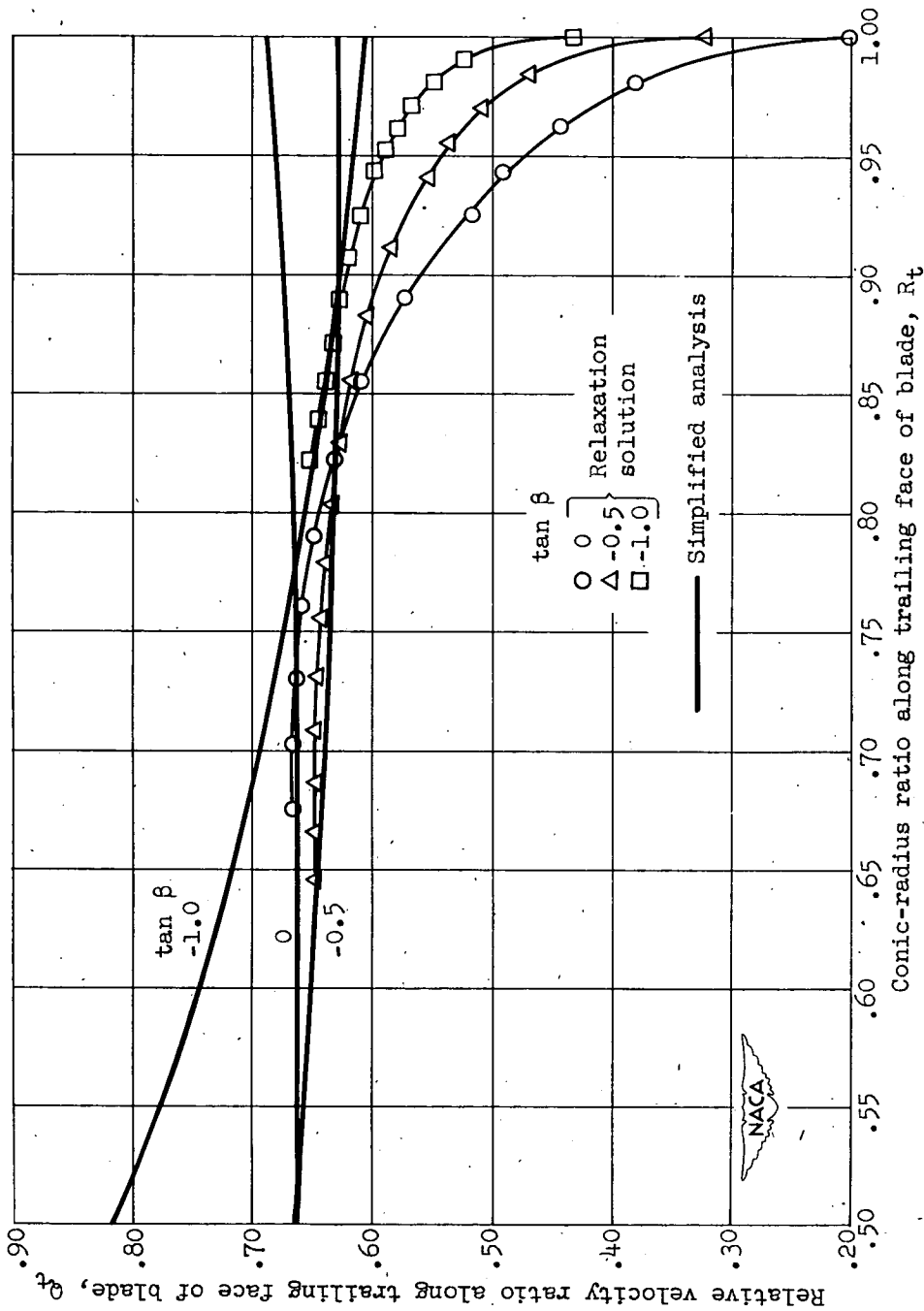


Figure 11. - Comparison of variation in relative velocity ratio along trailing face of blade with conic-radius ratio along trailing face of blade as obtained by relaxation methods and by the simplified analysis of three values of blade angle β . Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).

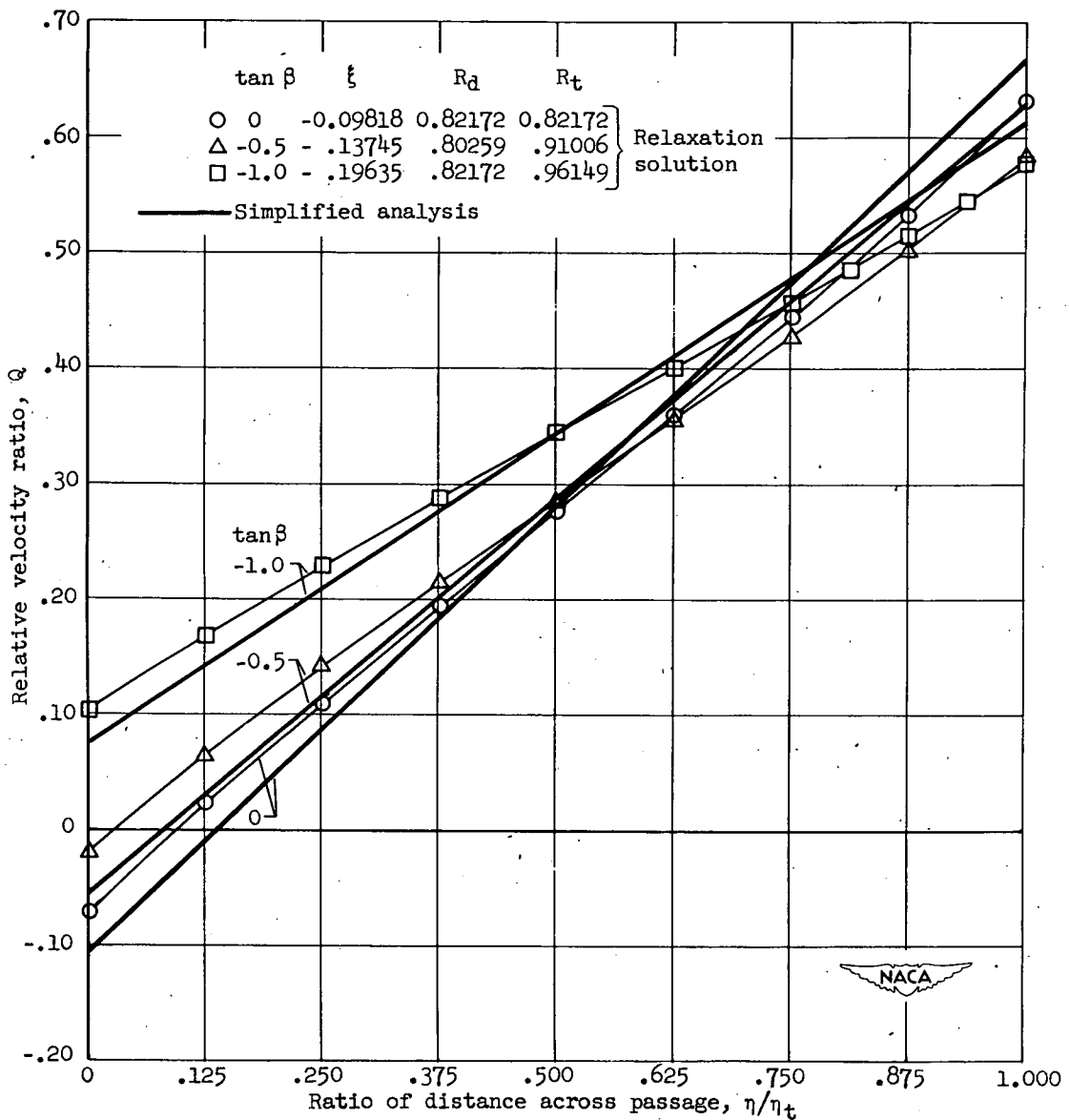


Figure 12. - Comparison of variation in relative velocity ratio with ratio of distance across passage (along lines of constant ξ) as obtained by relaxation methods and by the simplified analysis for three values of blade angle β . Flow coefficient ϕ , 0.5; impeller-tip Mach number M_T , 1.5; exponent m (for variation in H with R), -1.0; included passage angle between blades σ , 18° ; compressible flow ($\gamma = 1.4$).

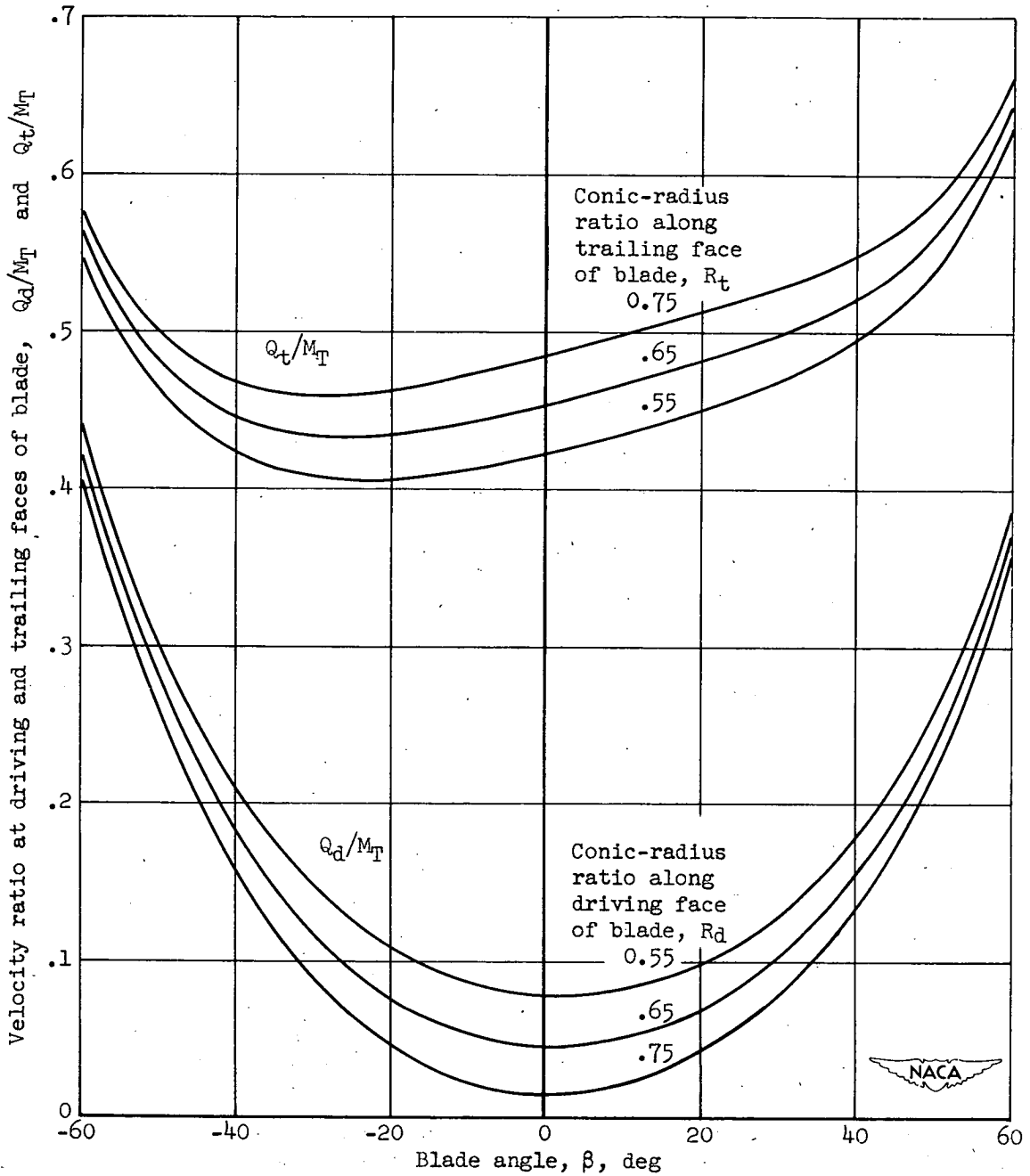


Figure 13. - Variation in velocity ratio at driving and trailing faces of blade with blade angle for several values of conic-radius ratio. Ratio ϕ/M_T , 0.250; exponent m (for variation in H with R), -1.0; included angle between blades across the passage σ_T , 18° ; incompressible flow.